Functions

Function Notation

- **Wildcard Variables to define functions**
- **Declaring Names and changing a declared name**
- **Using PreDefined functions**

Functions take three forms in *MathView*: explicitly defined, User Defined, and PreDefined. You choose the form, depending on the scope of your problem.

Explicit Functions

You create explicitly defined functions by typing or manipulating an equation into functional form. For example, to graph the equation below, you isolate the dependent variable from all independent variables. You do this by selecting the dependent variable ***y*** and choosing the **Isolate** command from the menu, clicking on the Palette icon, or by moving ***y*** with the hand to the Prop icon.

$$-2x = x^2 - y + 12$$

$$y = x^2 + 2x + 12 \quad \textit{Isolate} \longleftarrow \text{explicit function}$$

MathView also allows you to create custom “User Defined” functions. You input these functions with Wildcard variables (page 64).

User Defined Functions

You enter User Defined functions by using the function notation ***f*(*x*)**, where the independent variables are input as Wildcards. To input a variable as a Wildcard, choose the letter from the Wildcard palette or type a ? prior to typing the letter.

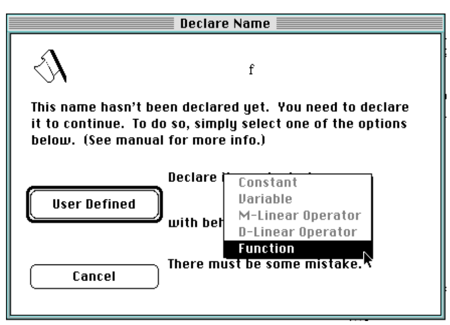
- Input: `f(?x)=?x^2+3*?x-33`

$$f(x) = x^2 + 3x - 33$$

The function ***f*** is now defined, but you cannot use it until you declare it as a function.

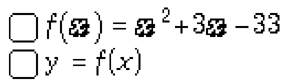
You do not have to define the function at this time, as *MathView* will ask you to define ***f*** the first time you try to use it.

- Declare ***f*** at this time by choosing **Clarify** from the **Notebook** menu. The dialog box below will open. Choose **Function** under the Pop-up menu and click **User Defined**, or press the Return key (return).

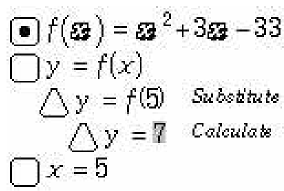


MathView tries to guess the class of the name you choose. In this case, *MathView* guessed that you wanted ***f*** to be a function, therefore the dialog defaulted to **Function**.

- To use the new function, input the equation **y = f(x)**, this time without using Wildcard variables.



- Input a value for x, substitute this equation into the $y = f(x)$ equation, and **Calculate** the RHS for a value.

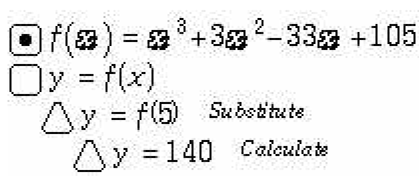


Substitute the ***x*** **=** 5 equation into the ***y = f(x)*** Prop and **Calculate** the RHS.

A nice feature of *MathView* is that you can collapse cascades, hiding intermediate steps. In the example to the right, select the final conclusion (***y***=140) and **Notebook ▶ Indent Left** until it is all the way to the left. Now double-click on the ***y=f(x)*** Prop icon. You can now change the ***x***=5 Prop and see the result without seeing the intermediate steps.

MathView has dynamically linked the two equations, now that you have defined the function and have made a substitution. Any change to the substitution equation or to the function itself now changes the Conclusion. To demonstrate, change $x = 5$ to $x = 3$, or change the function (the method below).

- Change the function to the following. Notice how *MathView* displays a hatching across the old conclusion as you change the function. This hatching symbolizes that you are changing the Theory.



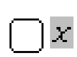
New Function and Result.

You can enter the function by itself without the ***y*** =, creating an expression rather than an equation. Select the expression and **Calculate**. Since you have defined the function before using it in this example, **f(x)** displays on the Functions palette, allowing you to click on its image to input the Assumption.

Do not confuse the function **f(x)**, which will show up on the Functions palette along with the other defined functions, with the large ***f(x)*** which toggles back and forth between the Functions and Variable palettes.

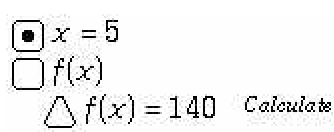
- Input x below the $x = 5$ Prop, select x, and click **f(x)** on the palette.

Select ***x***

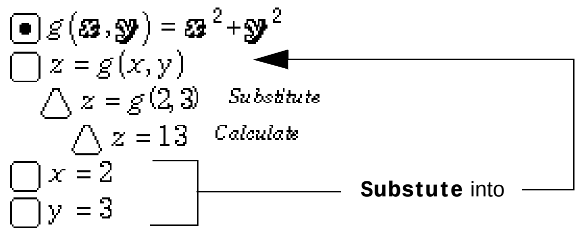


and click on palette image of f(x)

Select ***f(x)*** Prop and **Calculate**



You can use more than one independent variable to create an equation to plot three-dimensional objects or to study functions with several variables. You separate the parameters of the function with a comma.

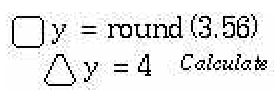


PreDefined Functions

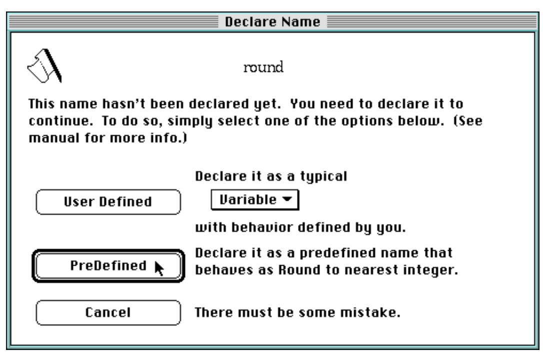
MathView has approximately 47 pre-defined functions and 16 pre-defined names, each with a behavior consistent with mathematical convention. The Function palette in the standard notebook contains 16 of these functions. Each is available at the click of the mouse.

You can create both User Defined and PreDefined functions by first selecting **Name Decl.** under the **Notebook ▶ Insert** menu. See page 63.

You use a PreDefined function by merely typing its name in your notebook. *MathView* will ask you to declare it the first time you use it or when you **Clarify** the notebook. For example, *MathView* has a predefined numeric function that rounds a number to the nearest integer, called **round** . The following example initiates the **Declare Name** dialog box, asking you to declare **round**. *MathView* presumes that you want the function **round**, but allows you the option of declaring it a User Defined name. If you click on the default **PreDefined** button, the function will act as you would normally expect (see below). By using this function once in the Notebook, it will be available to you in subsequent manipulations and the name **round** will show up on the Function Palette.

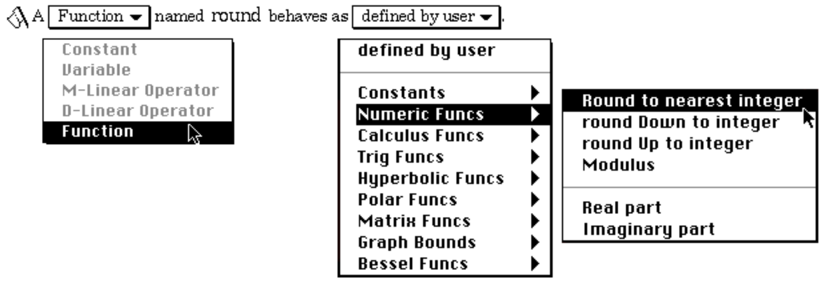


When you select the RHS and choose Calculate...



this dialog box appears.

You cannot use **round** to round a number if you have previously defined the name "round" as a User Defined variable. If you have done this by mistake, select the **round** definition nested inside the Declaration Prop (double-click on the first Prop in your notebook) and change the declaration. You do this by choosing the appropriate definitions under the Pop-up menus. Change **Variable ▼** to **Function ▼**, and **defined by user ▼** to **Numeric Funcs ▶ Round to nearest integer ▼**.


A Function named round behaves as defined by user.
Constant
Variable
M-Linear Operator
D-Linear Operator
Function
defined by user
Constants
Numeric Funcs
Calculus Funcs
Trig Funcs
Hyperbolic Funcs
Polar Funcs
Matrix Funcs
Graph Bounds
Bessel Funcs
Round to nearest integer
round Down to integer
round Up to integer
Modulus
Real part
Imaginary part

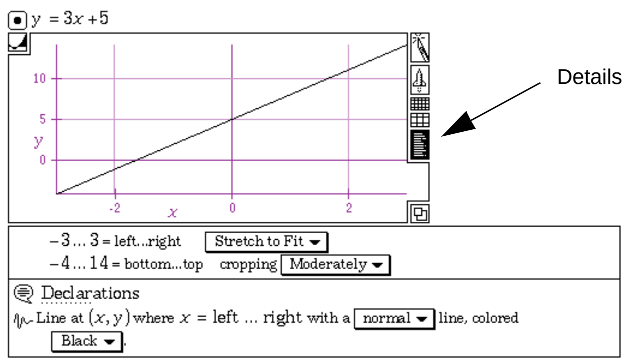
Linear Functions

- Plotting points in a Linear Graph Theory
- Indexing variables
- Customizing line details

Linear functions are normally the first functions you study but, as you well know, they become important components of more advanced subjects. An example is the linearization of curves. In this section, linear functions help introduce the dynamic aspect of Graph Theories, the indexing of variables, and how you can plot single points.

- Input the equation **y = 3x+5** in its own Prop.

- With the cursor located anywhere inside the function, select **Linear** under the **Graph** menu or click on the Palette image. *MathView* will assign a domain to the plot and adjust the Viewport according to its internal rules.

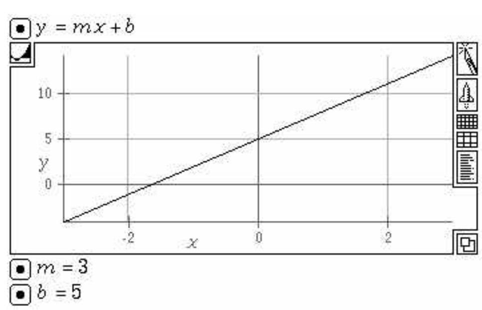


Details open

The standard Graph Theory Declarations and a Line Plot detail appear inside the graph details area. The important item to note here is that the detail denotes the line as, **Line at** (***x,y***). Even though you have defined ***y*** outside the Graph Theory, you can change the expression $3x + 5$ and the plot will adjust accordingly.

You can use *MathView* to study the effect on the slope and ***y***-intercept by giving both variable names.

- Set up your Notebook in the following manner. To observe the effects of the slope and ***y***-intercept on the function, change the values given to ***m*** and ***b***.



Declare ***m*** as a User Defined variable. ***b*** is defined as a constant, by default.

Change ***m*** and ***b***.

Slope

Given two points, you define the slope "***m***" of the line containing both as

$$m = slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Below is a Theory where you calculate the slope of the equation given the points (2,3) and (5,8).

- Using subscripted variables, define each point as shown below. Select the RHS of the slope equation by clicking once on the fraction line, and **Calculate** to obtain the value for ***m***.

$x_1 = 2 \quad y_1 = 3$

$x_2 = 5 \quad y_2 = 8$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Result

$m = 1.6667$ *Calculate*

To determine the equation for the line containing these points, solve the general equation for a line in Point Slope form for ***y***.

Notice that the Point-Slope equation only contains one subscripted variable at this time.

Remember to hold down the Shift key to select

- Re-input the equation using only the first point. Substitute the values determined in the conclusions above (x_1, y_1, and ***m***) into the line equation, and **Isolate** ***y***. **Expand** the RHS of the resulting equation for the answer.

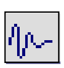
$m = \dfrac{y - y_1}{x - x_1}$ ← Equation in Point-slope form using the first point (x_1 , y_1)

$1.6667 = \dfrac{y - 3}{x - 2}$ *Substitute*

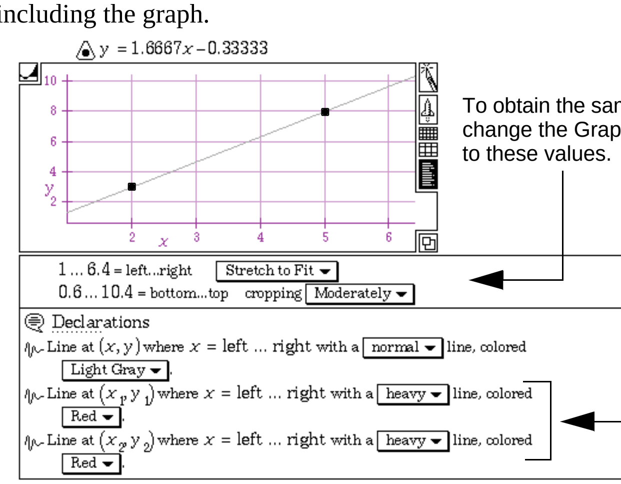
$y = 1.6667(x - 2) + 3$ *Isolate*

$y = 1.6667x - 0.33333$ *Expand* Plot this equation

You can now plot this equation along with the given points.



- Select the last equation and generate a linear graph.
- Add two line plots and re-define their details as (x_1 , y_1) and (x_2 , y_2) respectively. Choose a different color and pick **heavy**. This will help make the points show up. You can change the values of the points in the first four Props (the subscripted variables) and watch the whole Theory change, including the graph.



To obtain the same graph change the Graph Bounds to these values.

Details defining points

Quadratic Functions

- The Apply, Move Over, and Factor commands
- UnDo and UnCalculate
- Square Root Op
- Zooming in on a graph to find graph roots

This section demonstrates four methods of solving quadratic functions: Factoring, Completing the Square, the Quadratic Formula, and Reading Roots off of a graph.

Use the following equation for the examples in this section.

$$2x^2+8x=6$$

Factoring

- Place the equation into standard form. You do this by subtracting 6 from each side (adding a negative 6). Use the Hand to move the 6 over to the LHS (left hand side of the equation), or **Apply** (+ – 6) to each side.

Using Hand to **Move Over**

$$2x^2+8x-6$$

$$2x^2+8x=6$$
$$2x^2+8x-6=0 \quad \textit{Move Over}$$

Using **Apply** to add **(-6)** to each side

$$2x^2+8x=6$$
$$2x^2+8x=6$$
$$(2x^2+8x)+(-6)=6+(-6)$$
$$2x^2+8x-6=0 \quad \textit{Simplify}$$

- Select the LHS and **Factor**, using the method of your choice.

$$2x^2+8x-6=0$$
$$2(x+\sqrt{7}+2)(x-\sqrt{7}+2)=0 \quad \textit{Factor}$$

Factor the LHS

You can let *MathView* do all the work by selecting both ***x***s on the LHS (Shift-click) and choosing **Isolate** with **Auto Casing** turned on.

- One at a time, select the **x**s in the result above and **Isolate** for the answers (not shown).

Completing the Square

You can use the method of Completing the Square to solve the equation.

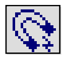
- Follow the instructions on the right.

$$2x^2+8x=6$$
$$\frac{2x^2+8x}{2}=\frac{6}{2} \quad \textit{Apply}$$
$$\tfrac{1}{2}(2x^2+8x)=3 \quad \textit{Simplify}$$
$$x^2+4x=3 \quad \textit{Expand}$$

Apply and divide by the coefficient of x^2.

Simplify and **Expand** the result.

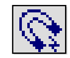
- Using **Apply** a second time, add the square of one half the coefficient of x to both sides. **Simplify** the result and **Factor** the LHS.



$\triangle\ (x^2+4x)+2^2 = 3+2^2$ *Apply*

$\triangle\ x^2+4x+4 = 7$ *Simplify* ← Select the LHS and **Factor**.

$\triangle\ (x+2)^2 = 7$ *Factor*



- **Apply** the Square Root Op to each side and solve for x with **Auto Casing** turned on.

Alternatively, you can select the x in the equation $(x+2)^2 = 7$ and **Isolate** with **Auto Casing** turned on.

$\triangle\ (x+2)^2 = 7$ *Factor* — **Apply** the Square Root Op.

$\triangle\ \sqrt{(x+2)^2} = \sqrt{7}$ *Apply*

$\triangle\ \sqrt{(x+2)^2} = \sqrt{7}$ *Apply* — **Isolate** x with **Auto Casing** ON.

$\triangle\ \sqrt{(x+2)^2} = \sqrt{7}$ *Apply*

$\bigcirc\ \triangle\ x = \sqrt{7}-2$ *Isolate* $\quad \bigcirc\ \triangle\ x = -\sqrt{7}-2$ *Isolate*

Quadratic Formula

The roots of the Quadratic Equation $ax^2+bx+c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

Another method you can use is to select both xs in the original equation with a shift-click and **Isolate** with **Auto Casing** turned on.

- Enter the coefficients of the given equation in their own Props.
- **Substitute** these into two Props set up with the Quadratic Formula.
- **Expand** for the answer.

$\square\ 2x^2+8x-6 = 0$ — Define coefficients in separate Props.

$\square\ a = 2 \quad \square\ b = 8 \quad \square\ c = -6$

$x = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$

$\triangle\ x = \frac{1}{4}(\sqrt{112}-8)$ *Substitute*

$\triangle\ x = \sqrt{7}-2$ *Expand*

$\square\ a = 2 \quad \square\ b = 8 \quad \square\ c = -6$

$x = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

$\triangle\ x = \frac{1}{4}(-\sqrt{112}-8)$ *Substitute*

$\triangle\ x = -\sqrt{7}-2$ *Expand*

Reading Roots off of a Graph

Notice that *MathView* generates a new Assumption Prop. This result happens anytime you change a Conclusion. Editing an Assumption, on the other hand, does not generate a new Assumption.

You must plot the equation to use the built-in root finding facility. Since the given equation is not in functional form you must first manipulate it.

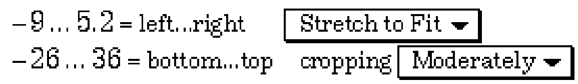
- Isolate the zero on the RHS and change to y.

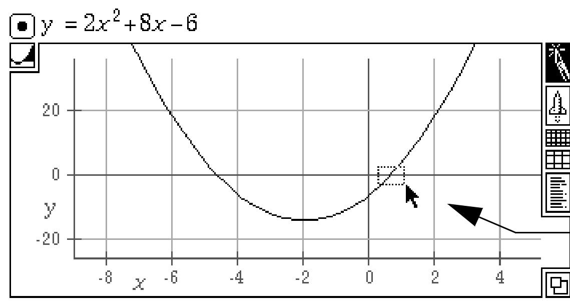
$2x^2+8x-6 = 0$

$\triangle\ 0 = 2x^2+8x-6$ *Isolate* — Select **0** and type y.

$\square\ y = 2x^2+8x-6$ ← New assumption generated.

- Place the cursor inside this new Assumption and generate a Linear Graph Theory. Adjust the Viewport by changing the details to the following.

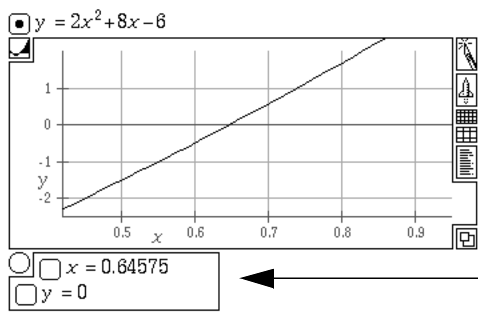




Zooming in on the positive root.

- You must zoom in on each ***x***-intercept to find the roots. Use the knife to do the zoom on the positive crossing (see above).
- Make sure only one intercept is in view in the Graph Theory and choose **Find Graph Root** under the **Manipulate** menu. *MathView* displays the root in its own Case Theory just below the graph.

MathView will sometimes ask you to to zoom-in closer, even though only one crossing is showing in the Viewport.



Zoom in close on an ***x***-axis crossing.

Select **Find Graph Root** under the **Manipulate** menu.

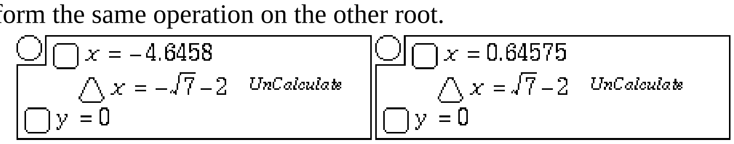
Root displayed in Case Theory.

Undo 

- Before performing any other manipulation, choose **Undo** under the **Edit** menu. You are now back to the graph's original view.

Notice that the Case Theory remains intact and the *root is in decimal form.*

- Select the number and choose **UnCalculate** (found in **Other** under the **Manipulate** menu) or click on the Palette icon found in the small hammer sub-palette).
- Perform the same operation on the other root.



Note two items here. First, if there is more than one Graph Theory in the Notebook, you will need to select the graph first, so *MathView* will know which graph to use. Click on the icon in the upper left corner of the Graph (the whole graph will highlight).

Second, you must zoom in very close to the zero crossing to have *MathView* find the precise value, $y = 0$. Many times ***y*** will equal a very small number very close to zero. You can sometimes use **UnCalculate** to force $y = 0$.

Polynomial Functions

- User Defined variables
- How to substitute more than one variable at a time into an equation

The multiple coefficients in polynomial functions reinforce the flexibility *MathView* provides by allowing you to control variables outside Graph Theories.

Set up a *MathView* notebook in the following manner to observe the effects of changing the coefficients of a polynomial function. Use the following third degree equation for this example.

$$y = 2x^3 - 9x^2 - 12x + 35$$

Remember to press the space bar or type an * to denote a multiplication between the coefficients and the variables.

- Define each coefficient as a separate variable in four separate Props (***A***, ***B***, ***C***, and ***D***, respectively).

$$y = Ax^3 + Bx^2 + Cx + D$$

$A = 2$

$B = -9$

$C = -12$

$D = 35$

Declare each coefficient a **User Defined Variable**.

- Select all of the variables by clicking on their equal signs while holding down the Shift key.
- **Substitute** into the first Prop with the Hand.
- After *MathView* displays the conclusion, generate a linear Graph Theory using this equation. Make sure the cursor is somewhere inside the new conclusion Prop before you select **Graph ▶ y=f(x) ▶ Linear**.

If you want, you can skip the substitution and merely select and graph the equation in the first Prop. This action generates the same plot, but uses different Working Statements.

$y = Ax^3 + Bx^2 + Cx + D$

$y = 2x^3 - 9x^2 - 12x + 35$ *Substitute* ← Plot this equation.

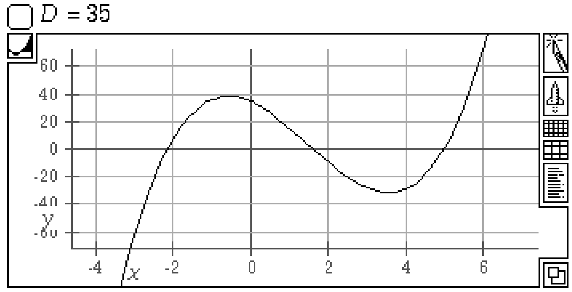
$A = 2$

$B = -9$

$C = -12$

$D = 35$

When requested, define the axes as ***x*** and ***y***, respectively.



$-4.6 \ldots 7.4$ = left...right Stretch to Fit

$-72 \ldots 72$ = bottom...top cropping Moderately

Graph bounds of this plot

You may have adjust the graph depending on the coefficients you choose.

- Change the values of each of the coefficients and observe the effects. Try setting every coefficient equal to zero, and change one value to observe what happens. Then, with this one variable set to a value which best shows its form, change each of the others to observe their effects on the plot.
- Below is the Theory showing the example you used in the previous section on Quadratic Functions. See page 104.

$y = Ax^3 + Bx^2 + Cx + D$

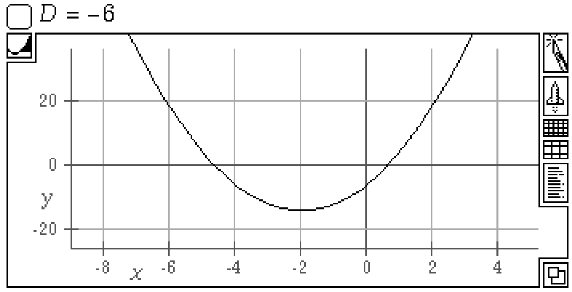
$y = 2x^2 + 8x - 6$ Substitute

$A = 0$

$B = 2$

$C = 8$

$D = -6$



Teacher's Note

The example above is an excellent one to use to introduce Taylor Polynomials. In a lab setting with groups of two or three, have the students add the sine function to the Graph Theory. In addition, have them add two more terms to the general equation.

$y = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$

Now, challenge them to change the coefficients so that the polynomial fits the sine curve.

Another good use for this method is to study the parabola using the general form,

$$y = ax^2 + bx + c, \quad a \neq 0.$$

Using the discriminant $b^2 - 4ac$, you can study the equations for the ***x***-axis and the ***y***-coordinate. In addition, you can use the general equation,

$y = a(x - h)^2 + k$, to study translations.

Set up a new notebook using the general equation or use a case theory if more than one graph is in the notebook.

$y = a(x - h)^2 + k$

$a = 1$

$h = 0$

$k = 0$

Plot ***y*** and change the parameters.

Lastly, you may want to set up a Graph Theory with the equation which generated the Plot hidden so you can direct students to "guess" the function. Make the equation a daughter Prop (page 33) of the graph that the equation generated. Then, double-click on the Graph icon to hide the equation, and choose **Notebook ► Lock Proposition**. This action locks the equation in a collapsed, hidden, state.

Now create a new equation with a different dependent variable and choose **Graph ► Additional ► Add Line Plot**. Ask your students to mirror the original plot by changing the RHS of this new equation.

Rational Functions

- Animation
- The use of Case Theories to observe more than one plot
- Manually defining Viewport details

You write a Rational Function as a quotient of two polynomial functions.

$$f(x) = \frac{g(x)}{h(x)}, \quad h(x) \neq 0$$

Analyzing the various translations of rational functions provides an excellent opportunity to demonstrate the use of *MathView's* Case Theories. The example that follows will show you how to set up a Theory in which multiple graphs can use the same parameters to display more than one translation at a time.

You can find the asymptotes of rational functions easily by using *MathView* to factor or plot the denominator of the function. In the final subject in this section, you will use a similar method to decompose a complicated fraction into its parts.

Rational Translations

The very simple function $y = f(x) = 1/x$ is the basis for the following rational function. Set up your notebook in the following manner. Choose **Case Theory** in the **Insert** sub-menu under **Notebook** to generate a Case Theory. The general equations you will use are

$$y = C\frac{1}{x^A + B} + D \qquad \text{and} \qquad y = C\frac{1}{-x^A + B} + D$$

To move the second Case Theory from below, where it will originally display, to the right of the first one, select the Prop by clicking on the circle icon, and with the Hand, move to the right of the box outlining the Case Theory. For help, see page 33.

- In side by side Case Theories, input the two equations.

$$y = C\frac{1}{x^A + B} + D \qquad y = C\frac{1}{-x^A + B} + D$$

- Outside of the Case Theories described above, define the User Defined variables, each in its own Prop. You can locate these Props above or below the Case Theories. They are outside of the Case Theories, so changes to their values will affect variables inside both Case Theories at the same time.

$A = 1$
$B = 0$
$C = 1$
$D = 0$

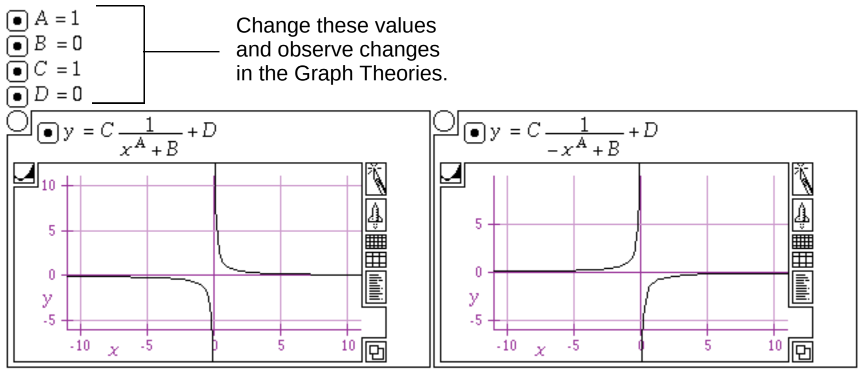
Declare each a **User Defined Variable**.

- Select each of the two main equations and generate a **Linear** Graph Theory. Change the Viewport details of both graphs to the values shown below.

−11 ... 11 = left...right   Stretch to Fit
−6 ... 11 = bottom...top   cropping Moderately

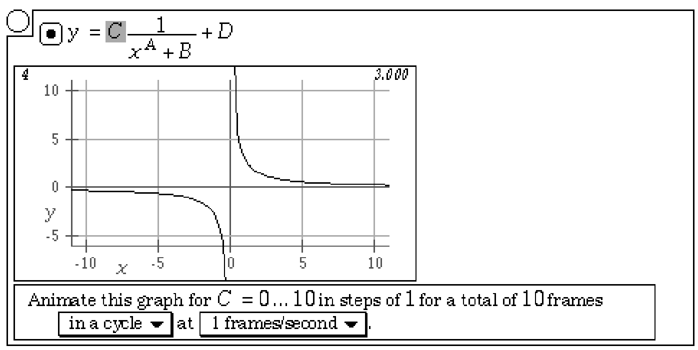
You do not have to substitute the variables into the equations. The equations inside the Case Theories "look outside" to find values when plotted.

Your Notebook should now look like the following.



Change the various parameters and watch the effects ripple through both Case Theories.

You can animate the parameters in the example above (one at a time). Below is the left graph, from above, with **C** selected and animated (with values from **0** to **10** at **1** frame per second). The screen shot below captured the fourth frame, in which **C = 3.0**.



You can use a parameter outside the Graph Theory as long as you select the graph along with the parameter. Shift-click the parameter and the graph. Now **Animate**.

You will find it easier to select a parameter inside one of the Case Theories to animate. *MathView* can only animate one graph at a time, and will choose the first graph in the notebook if you wish to animate the graph by selecting a variable in the list outside of the Case Theory.

Finding Asymptotes

Many times you can find the asymptotes of simple rational functions by inspection, but what happens when the function is complex?

$$y = \frac{-12x^3 - 18x^2 + 2784x + 8712}{x^5 - 105x^3 - 100x^2 + 2364x + 4320}$$

Use one of two *MathView* techniques to find asymptotes of more complicated rational functions. The first technique is to **Factor** the denominator and set each factor equal to zero. In the second, you pull out each expression, set it equal to ***y***, and plot the equation. You then find the roots in the Graph Theory. The first technique is below.

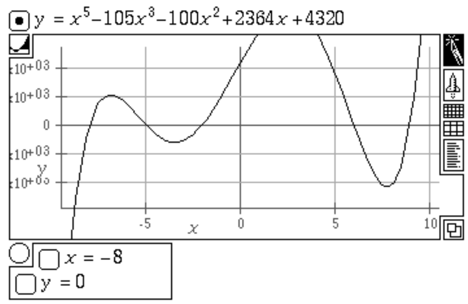
- Select the denominator by double-clicking on one of the operators (+ or –). Choose **Factor** under the **Manipulate** menu, or click on the Palette icon for the answer.

$$y = \frac{-12x^3 - 18x^2 + 2784x + 8712}{x^5 - 105x^3 - 100x^2 + 2364x + 4320} \quad \text{Select denominator}$$

$$y = \frac{-12x^3 - 18x^2 + 2784x + 8712}{(x-9)(x+8)(x-6)(x+5)(x+2)} \quad \textit{Factor}$$

The second technique requires you to create a new equation defining ***y*** equal to the denominator.

- Input the equation into a new Prop and plot. Zoom in on the five ***x***-intercepts, as you did on page 106, to find the roots. Shown below is the plot along with the first root.



Partial Fractions

If the degree of the denominator of a rational polynomial function is greater than the degree of the numerator, and if you can factor the denominator into non-repeating linear factors, you can write the original fraction as the sum of simpler fractions. This technique is important when studying integration where the integrand is one of these functions. See page 177 for an example.

This operation is the reverse of finding a common denominator of several fractions that you add together. You can learn several methods by hand, but with *MathView* at your command, you can use the power of the computer to automate the procedure.

- Enter the same function you used in the prior example, select the denominator, and **Factor**.

$$y = \frac{-12x^3 - 18x^2 + 2784x + 8712}{x^5 - 105x^3 - 100x^2 + 2364x + 4320}$$

$$y = \frac{-12x^3 - 18x^2 + 2784x + 8712}{(x-9)(x+8)(x-6)(x+5)(x+2)} \quad \textit{Factor}$$

- Select the RHS, by clicking on the fraction line, and **Expand**.

$$y = \frac{-12x^3 - 18x^2 + 2784x + 8712}{(x-9)(x+8)(x-6)(x+5)(x+2)} \quad \textit{Factor}$$

$$y = 3\frac{1}{x-9} - 2\frac{1}{x+8} - 6\frac{1}{x-6} + 3\frac{1}{x+5} + 2\frac{1}{x+2} \quad \textit{Expand}$$

You can also use a semi-manual method to demonstrate the mathematical concept.

- First input the following new expression and **Factor** the denominator.

$$\square\ \frac{6x^2+6x-6}{x^3+2x^2-5x-6}$$

$$\triangle\ \frac{6x^2+6x-6}{x^3+2x^2-5x-6}=\frac{6x^2+6x-6}{(x+3)(x-2)(x+1)} \quad \textit{Factor}$$

- In a new Prop re-input as follows.

$$\square\ \frac{6x^2+6x-6}{(x+3)(x-2)(x+1)}=\frac{A}{x+3}+\frac{B}{x-2}+\frac{C}{x+1}$$

- Manually take the denominator on the LHS times both sides (each term on RHS) and simplify the results.

$$\square\ \frac{6x^2+6x-6}{(x+3(x-2)(x+1)}\frac{(x+3)(x-2)(x+1)}{1}= \ldots\ldots$$

$$\ldots\ldots\frac{A}{x+3}\frac{(x+3)(x-2)(x+1)}{1}+\frac{B}{x-2}\frac{(x+3)(x-2)(x+1)}{1}+\frac{C}{x+1}\frac{(x+3)(x-2)(x+1)}{1}$$

$$\triangle\ 6x^2+6x-6=(x-2)(x+1)A+(x+3)(x+1)B+(x+3)(x-2)C \quad \textit{Simplify}$$

- Make a Prop giving a value to ***x*** so that one of the factors on the RHS goes to zero. Give ***x*** the value of – 1 so both ***A*** and ***B*** will go to zero.
- **Substitute** and solve for the remaining variable (***C*** in this case).

$$\triangle\ 6x^2+6x-6=(x-2)(x+1)A+(x+3)(x+1)B+(x+3)(x-2)C \quad \textit{Simplify}$$

$$\triangle\ -6=-6C \quad \textit{Substitute}$$

$$\triangle\ C=1 \quad \textit{Isolate}$$

$$\square\ x=-1$$

- Copy the result to a new Prop and change ***x*** to make the other factors go to zero and solve for ***A*** and ***B***. Substitute these into the original equation for the answer (shown below).

$$\square\ \frac{6x^2+6x-6}{(x+3)(x-2)(x+1)}=\frac{3}{x+3}+\frac{2}{x-2}+\frac{1}{x+1}$$

Teacher's Note

The example above, along with others in this guide, is reverse engineered. *MathView* makes this a very easy task and can help you create examples and test questions which serve your academic purposes.

Taking the first example under Partial Fractions, first enter the desired result.

$$\square\ \frac{3}{x-9}-\frac{2}{x+8}-\frac{6}{x-6}+\frac{3}{x+5}+\frac{2}{x+2}$$

Select the whole expression and select the **Collect** manipulation. This creates a very large expression not shown here. Now **Expand** the numerator.

$$=\frac{-12x^3-18x^2+2784x+8712}{(x-9)(x+8)(x-6)(x+5)(x+2)} \quad \textit{Expand}$$

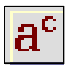
Finally, **Expand** the denominator to obtain the final result.

$$=\frac{-12x^3-18x^2+2784x+8712}{x^5-105x^3-100x^2+2364x+4320} \quad \textit{Expand}$$

Exponents & Logarithms

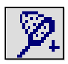
- Auto Simplify and its effect on exponents
- How scientific notation works in
- Subscripts and Superscripts

Before you can effectively use exponents and logarithms in *MathView*, you must become familiar with their input. You must also understand the effect that **Auto Simplify** and **Auto Casing** have on manipulations containing exponents.



In *MathView*, you enter superscripted indexes by using either the palette shortcut or by using the conventional ^ from the keyboard (shift-6). Complete by typing the exponent. To come out of the exponent back to the baseline, press esc.

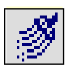
With the judicious use of the **Auto Simplify** command, you can demonstrate the various properties of exponents. For example, to generate the factors of x^5 , you select the expression, turn off **Auto Simplify**, and perform a **Collect**.



□ x^5

△ $x^5 = x\,x\,x\,x\,x$ *Collect*

Turn OFF **Auto Simplify,** select, and **Collect**.



To reverse the process, select the RHS of this new Prop and **Simplify**.

You toggle Auto Simplify on or off by selecting **Auto Simplify** in **Manipulation Prefs**, under the **Manipulate** menu.

Positive Integer Exponents

When n is a positive integer, b^n means that you have multiplied n factors of b together. You use this definition to find an equivalent form for an expression like

$$(b^4)(b^6) = (b \;\; b \;\; b \;\; b) \;\; (b \;\; b \;\; b \;\; b \;\; b \;\; b) = b^{10}$$

Generalizing this statement presents the first Property of Exponents:

$$(b^m)(b^n) = b^{m+n}$$

Table 2 below summarizes the Properties of Exponents using *MathView*. Noted on the right side of each manipulation is the state of Auto Simplify. A check mark indicates whether Auto Simplify is on or off. In some cases, the manipulation will work while *MathView* is in either state. For the following manipulations to work properly, declare each parameter a **Constant**.

Table 2: Properties of Exponents

Property #, Selection, and Manipulation	**Auto Simplify** ON	**Auto Simplify** OFF
#1 □ $b^m b^n$ △ $b^m b^n = b^{m+n}$ *Simplify*	✔	✔

Table 2: Properties of Exponents

Property #, Selection, and Manipulation	Auto Simplify ON	Auto Simplify OFF
Selection: b^{m+n} $b^{m+n} = b^m b^n$ *Collect*		✔
#2 Selection: $\frac{b^m}{b^n}$ $\frac{b^m}{b^n} = b^{m-n}$ *Simplify*	✔	
Selection: b^{m-n} $b^{m-n} = b^m b^{-n}$ *Collect* $b^{m-n} = b^m \frac{1}{b^n}$ *Simplify*	✔ (Simplify)	✔ (Collect)
#3 Selection: b^0 $b^0 = 1$ *Simplify*	✔	✔
#4 Selection: $(ab)^m$ $(ab)^m = a^m b^m$ *Expand*	✔	✔
Selection: $a^m b^m$ $a^m b^m = (ab)^m$ *Factor*	✔	✔
#5 Selection: $(b^m)^n$ $(b^m)^n = b^{nm}$ *Simplify*	✔	✔
Selection: b^{nm} $b^{nm} = (b^n)^m$ *Collect*		✔
#6 Selection: $\left(\frac{a}{b}\right)^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ *Expand*	✔	✔

Negative Integer Exponents

When the exponent is a negative integer then $b^{-n} = 1/b^n$.

Table 3: Negative Integer Exponents

Property & Manipulation	**Auto Simplify** ON	**Auto Simplify** OFF
□ b^{-n} △ $b^{-n} = \frac{1}{b^n}$ *Simplify*	✔	✔

Rational Number Exponents

What meaning does the symbol $b^{1/2}$ have? By definition, it is the square root of the base ***b***. According to Property #5, the following holds true.

$$(b^{1/2})^2 = b^{(1/2)(2)} = b^1 = b$$

So $b^{1/2}$ is a solution to $x^2 = b$, which means that $b^{1/2} = \sqrt{b}$. This expression, by convention, is the Principal Square Root.

Table 4: Rational Number Exponents

Property & Manipulation	**Auto Simplify** ON	**Auto Simplify** OFF
□ $\left(b^{\frac{1}{2}}\right)^2$		
△ $\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2}}b^{\frac{1}{2}}$ *Collect*		✔
△ $\left(b^{\frac{1}{2}}\right)^2 = b$ *Simplify*	✔	✔
□ $b^{\frac{1}{2}}$ △ $b^{\frac{1}{2}} = \sqrt{b}$ *Simplify*	✔	✔
□ $\sqrt{b}$ △ $\sqrt{b} = b^{\frac{1}{2}}$ *Expand*		✔

MathView does not have an ***n***$^{\text{th}}$ root operator, so $b^{1/n}$ will not simplify to $\sqrt[n]{b}$.

Scientific Notation

Manipulating numbers in scientific notation requires you to be familiar with the definitions and properties of integral exponents. Although the symbolic capability of *MathView*, for the most part, eliminates the need to use this notation, it does allow you to input and manipulate numbers in scientific notation. You do this by controlling the notebook display precision. In addition, *MathView* displays some numbers in scientific notation regardless of the precision setting, particularly in Graph Theories when the labels become too large.

MathView gives you two methods of inputting numbers in scientific notation. The first method requires you to type the non-zero coefficient, e.g., **2.34**; to type * on the keyboard (or pressing the space bar); and then to type **10** and its exponent (i.e., **7**), delimited with a ^.

2.34 * 10 ^ 7

▢ $2.34 \cdot 10^7$ → ▢ $2.34 \cdot 10^7$

△ $2.34 \cdot 10^7 = 23400000$ *Expand*

An **Expand** will generate the number with all of its digits showing, regardless of the Display Precision setting (up to a maximum of 9 digits or the notebook setting, whichever is greater).

You may also click on the **e** on the Variables palette in the area containing the numbers 0 to 9.

The second method is to type **2.34 e 7**. Notice that you do not need to create a superscript to input the exponent. *MathView* automatically generates the times sign (X), the **10**, and an exponent place-holder.

Type: **2.34 e 7** → ▢ 2.34×10^7

You choose the display precision by selecting **Display Precision** under the **Notebook** menu. Since display precision is in the **Notebook** menu, it affects everything in the notebook. Thus you cannot choose to display one Prop in scientific notation and another in regular notation.

You do not effect the internally maintained precision by setting the display precision. *MathView* has a maximum display precision of 15 digits.

You can show the steps of simple arithmetic using scientific notation by turning **Auto Simplify** off. The example below demonstrates this concept by adding, multiplying, and dividing two numbers (***x*** and ***y***), with **Auto Simplify** turned off.

▢ $x = 1.1111 \times 10^{10}$

▢ $y = 2.2222 \times 10^{10}$

Substitute into Props below

▢ $x + y$

△ $x + y = 1.1111 \times 10^{10} + 2.2222 \times 10^{10}$ *Substitute*

△ $x + y = 3.3333 \times 10^{10}$ *Simplify*

▢ xy

△ $xy = 1.1111 \times 10^{10} \cdot 2.2222 \times 10^{10}$ *Substitute*

△ $xy = 2.4691 \times 10^{20}$ *Simplify*

▢ $\frac{x}{y}$

△ $\frac{x}{y} = \frac{1.1111 \times 10^{10}}{2.2222 \times 10^{10}}$ *Substitute*

△ $\frac{x}{y} = 0.5$ *Simplify*

Display Precision set to 5

Auto Simplify OFF

Common Logarithms

The common logarithm, base 10, is a PreDefined Function in *MathView*. You enter it by typing **log(**, or by clicking on the palette image **log(x)** . You generate numeric answers with a **Calculate** manipulation.

Enter a number, select it, and click on the palette function log(x).

☐ 25   Click on **log(x)** ⟶ ☐ $\log(25)$

or

Type **log(25** ⟶ ☐ $\log(25)$

☐ $\log(25)$
△ $\log(25) = 1.3979$ *Calculate*

Logarithm to base b

You enter logs to other bases by using a subscript. Type **log**, an underscore (shift—-), and the letter **b**. Then press esc and the left parentheses prior to entering the number. You can also type **log** and click on the subscript icon on the palette.

log _b esc **(7** ⟶ ☐ $\log_b(7)$

log a_b **b** Ÿ **(7** ⟶ ☐ $\log_b(7)$

Natural Logarithm

You enter the logarithm to the base ***e*** (page 118) as a log to the base ***b*** where ***b*** = ***e***, or you input it by using the PreDefined function **ln(x)**.

☐ $\log_e(7)$
△ $\log_e(7) = 1.95$ *Calculate*
☐ $\ln(7)$
△ $\ln(7) = 1.95$ *Calculate*

Properties of Logarithms

You can generate the Properties of Logarithms symbolically by performing an **Expand** with **Auto Simplify** turned ON.

☐ $\log_b(mn)$
△ $\log_b(mn) = \log_b(m) + \log_b(n)$ *Expand*
☐ $\log_b\left(\frac{m}{n}\right)$
△ $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$ *Expand*
☐ $\log_b(m^n)$
△ $\log_b(m^n) = n\log_b(m)$ *Expand*
☐ $\log_b(b^x)$
△ $\log_b(b^x) = x$ *Expand*

☐ $\ln(mn)$
△ $\ln(mn) = \ln(m) + \ln(n)$ *Expand*
☐ $\ln\left(\frac{m}{n}\right)$
△ $\ln\left(\frac{m}{n}\right) = \ln(m) - \ln(n)$ *Expand*
☐ $\ln(m^n)$
△ $\ln(m^n) = n\ln(m)$ *Expand*
☐ $\ln(e^x)$
△ $\ln(e^x) = x$ *Expand*

You evaluate logarithms by isolating the unknown variable. In the first example below, **Isolate** the exponent ***x*** with **Auto Simplify** turned off and **Simplify** the results. In the second example, first apply the log function to both sides. **Expand** the result, **Isolate** the variable, and **Calculate** the RHS.

#1 **Auto Simplify** → Off

☐ $2^x = 8$
△ $x = \log_2(8)$ *Isolate*
△ $x = 3$ *Simplify*

#2 **Auto Simplify** → On

☐ $2^x = 8$
△ $\log(2^x) = \log(8)$ *Apply*
△ $\log(2)x = \log(8)$ *Expand*
△ $x = \frac{\log(8)}{\log(2)}$ *Isolate*
△ $x = 3$ *Calculate*

Exponential & Logarithmic Functions

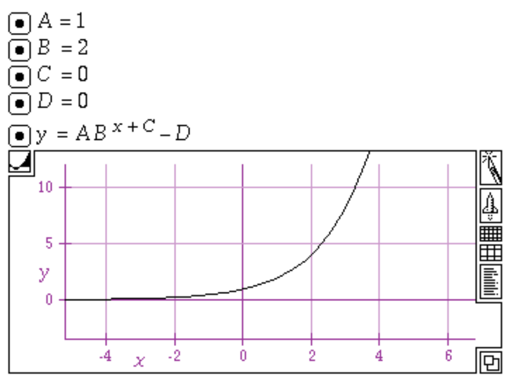
- Using Graph Theories to study Transformations
- ***e*** and the use of Tables to analyze limits
- Parametric Plots

The exponential and logarithmic functions are very interesting functions to analyze because of their relationship to each other, their importance in the study of calculus, and their connection to the scientific study of growth and decay.

In this section, you will use the same method described in the sections on Rational and Polynomial functions to study the exponential function and its transformations. You will use both linear graphs and tables to analyze the irrational number ***e***. Finally, you will create Parametric Graph Theories by plotting both the natural log and the natural exponential function in the same graph, thereby demonstrating their inverse relationship.

The Exponential Function and Its Transformations

In a New Notebook, or in a separate Case Theory, input the general exponential function b^x, expanded to show its transformations. Use capital letters for the transformation variables, so as to not conflict with any of *MathView's* pre-defined variables. Choose a base of 2 and generate a Linear Graph Theory.



Change each parameter to observe its affect. Of particular interest is the comparison between ***B*** between zero and one, and ***B*** greater than one. Animate ***B*** from 0 to 10 to see a very interesting process.

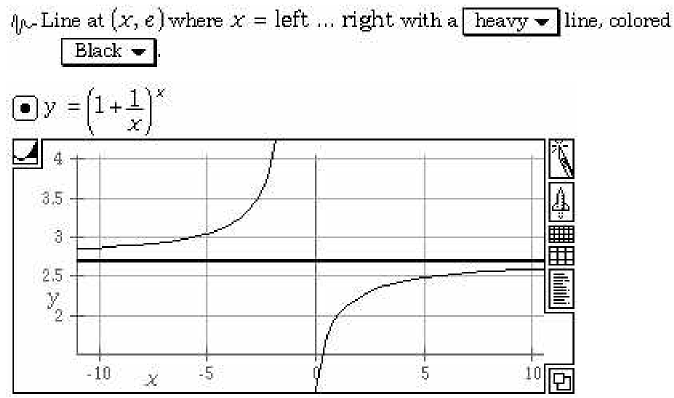
The Natural Exponential Function

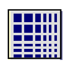
A particularly important function in the study of higher mathematics is that of the natural exponential function. Based on the irrational number $e \approx 2.7182818$, this function is just the regular exponential function using ***e*** as the base.

The following example shows you how plotting the function $y = (1 + 1/x)^x$ graphically displays a relationship which can help in the understanding of this important number.

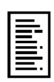
- Input the function and generate a Linear Graph Theory. After the plot generates, notice that for values of ***x*** approaching $\pm\infty$, the plot approaches the number ***e***.

- Add to this graph a new line plot and change the ***y*** to the constant ***e*** in the detail. The new detail and the resulting graph follows.

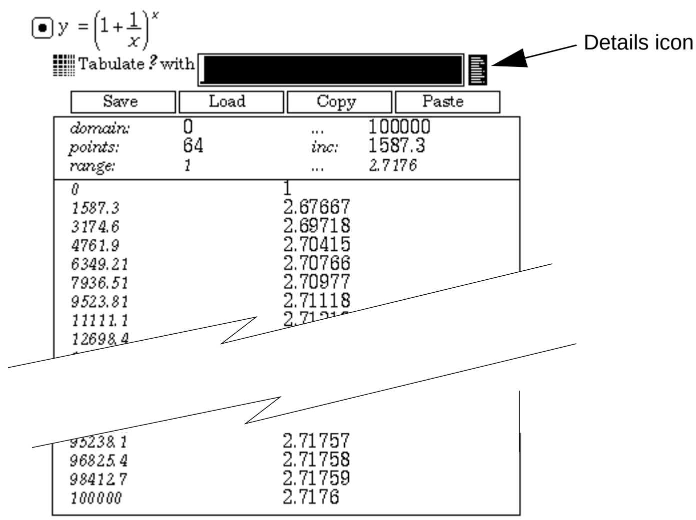




You can approximate the value for ***e*** by using this same function and *MathView's* Table generator.

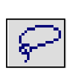


- Select the RHS of the equation above and choose **Table ▶ Generate...** under the **Manipulate** menu. Set the range for ***x*** from 0 to 10,000 and click **OK**. Open the table by clicking once on the Details icon and scroll to the end to see the value approach ***e***.

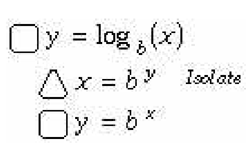


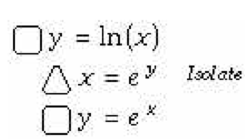
The Logarithmic Function

The inverse of the Exponential Function is the Logarithmic Function.



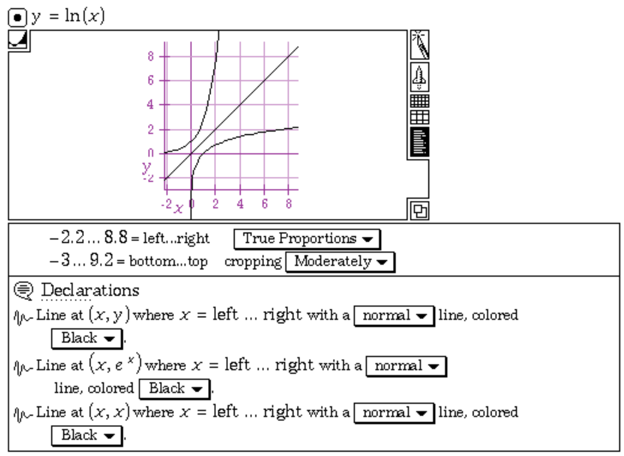
- To demonstrate this, input either the Common Log base ***b***, or the Natural Log function. Select the variable ***x*** and **Isolate**. Replace the ***x*** with a ***y*** and the ***y*** with an ***x***.





You can plot both in the same Graph Theory to demonstrate this relationship.

- Plot the Natural Log function.
- Add a line plot and change its detail so that it equals the exponential function.
- Add a third line defined as $y = x$. Use True Proportions.

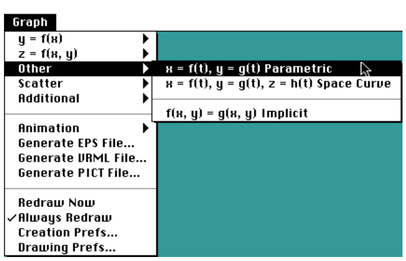


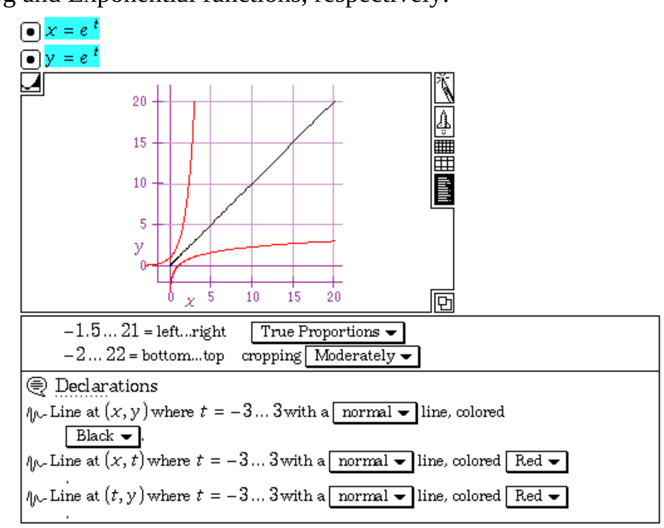
Both of the examples on this page require you to add two line plots. You do this by selecting the graph in question and choosing **Add Line Plot** from the **Graph ▶ Additional** menu, twice. You add two line details with parameters identical to the original plot. Change these details to the functions listed. Both examples show the details open to guide you.

MathView has a Parametric graph which can also demonstrate this relationship. To create a Parametric plot, you must select two equations of the form $x = f(t)$ and $y = g(t)$ at the same time before you plot. You do this by selecting one equation (click once on the equal sign), holding down the Shift key, and clicking on the other. After you have selected both, choose **Other ▶ x=f(t), y=g(t) Parametric** under the **Graph** menu. Define the axes as ***x*** and ***y***, and the parameter as ***t***.

You can demonstrate the inverse relationship of the two functions in the prior example by plotting ***x*** as a function of ***t*** in one Prop, and ***y*** as a function of ***t*** in the other. The resulting straight line verifies the inverse relationship.

- Input $x = e^t$ and $y = e^t$, select both equations, and plot. Add two line plots and define them as $y = t$ in one and $x = t$ in the other. These two additional plots are the Log and Exponential functions, respectively.





Trigonometric Functions

- Using PreDefined functions
- Creating your own Transformation Rule
- Creating your own User Defined function
- Polar and Rectangular coordinates

MathView uses Transformation Rules to implement trigonometric identities. In this section, you will learn how these rules work. You will also use the PreDefined functions, ToPolar and FromPolar, to create an interesting graph.

For this section, use *MathView's* distributed notebook called “Trig Functions”. You can find it in the *Transcendental Functions* Folder located inside the *Mathematics* Folder which comes with the program CD. Alternatively, you can copy the Trigonometric Declarations out of that notebook and paste them into your working notebook. See page 29 for instruction.

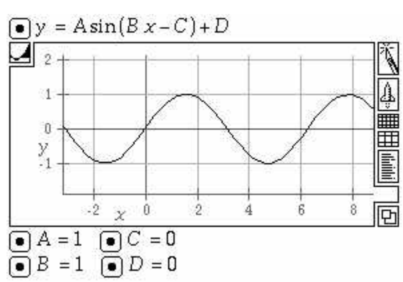
PreDefined Trig Functions

MathView has twelve PreDefined trigonometric functions which include the six normal functions and the six inverse functions. In addition, *MathView* has twelve hyperbolic functions and six polar functions.

All of these PreDefined functions work in the normal manner of all *MathView* functions. You use them by either selecting an argument and clicking on the function palette image or by typing the function name, an open parenthesis, and the argument. You plot a trigonometric function by creating a functional equation and generating a linear Graph Theory.

You can analyze trigonometric transformations by setting up your notebook in the following manner using User Defined variables.

- Set up the notebook with a function in Sinusoidal form and add to the Theory the variables ***A***, ***B***, ***C***, and ***D***. You can change, or animate each variable to study its effects.



Degrees vs. Radians

You can define the six basic trigonometric functions by using the ratio of the sides of a right triangle. These relationships provide information about the associated acute angle. This angle is in degrees which, in most cases, you must translate into radians.

MathView uses radians to represent all angle measurement, so to analyze angles given in degrees, you must first translate degrees into radians. You can have *MathView* transform a number from degrees to radians by using the PreDefined Name, degrees. To invoke this name, you enter a number representing degrees and attach to it the degree symbol. You do this by typing the number then *'o* (or on the Macintosh, [option]-o or [option] [shift] - 8). Select the Prop and **Calculate**. The result is in radians. Below 45 degrees is translated into radians.

After you define degrees, the degree symbol will display on the Variable palette.

- Enter **45 ' o** (this is o, not zero)
- Press the return key to accept the default, degrees, when the PreDefined Name declaration appears.
- Select and **Calculate**.

$$45°$$
$$45° = 0.7854 \quad \textit{Calculate}$$

To demonstrate the difference between degree and radians when you use them in manipulations, compare the sine of 45° to the sine of $\pi/4$ radians.

Sine of 45°

$$\sin(45°)$$
$$\sin(45°) = 0.70711 \quad \textit{Calculate}$$

Sine of π/4 radians

$$\sin\left(\frac{\pi}{4}\right)$$
$$\sin\left(\frac{\pi}{4}\right) = 0.70711 \quad \textit{Calculate}$$

Another way you can translate degrees into radians and also translate radians into degrees is to create two User Defined functions.

- Input the following User Defined functions. Remember, you must use Wildcard variables, and you must declare each name a function.

$$\text{DegToRad}(⊙) = \frac{\pi ⊙}{180}$$
$$\text{RadToDeg}(⊙) = \frac{180 ⊙}{\pi}$$

After you input, choose **Clarify** and declare both **User Defined** functions.

The names given to these two functions are just suggestions. You may choose any name you want. However, descriptive names are best.

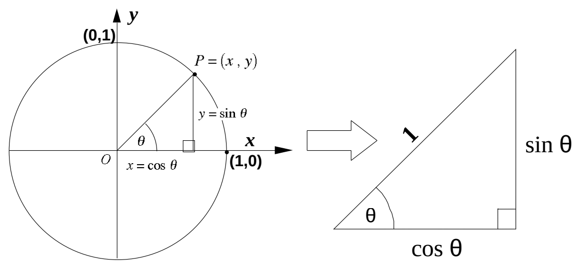
- To use the respective functions, click on their images in the Function palette where they will display after you declare them as functions.

$$\text{DegToRad}(45)$$
$$\text{DegToRad}(45) = 0.7854 \quad \textit{Calculate}$$
$$\text{RadToDeg}(0.7854)$$
$$\text{RadToDeg}(0.7854) = 45 \quad \textit{Calculate}$$

Trigonometric Identities

Because $\cos(\theta)$ and $\sin(\theta)$ are the ***x*** and ***y*** coordinates of a point on the unit circle, the following identity holds true.

$$(\cos(\theta))^2 + (\sin(\theta))^2 = 1$$



$$\tan\theta = \frac{\textit{Opposite side}}{\textit{Adjacent side}} = \frac{\sin\theta}{\cos\theta}$$

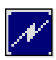
$$\cot\theta = \frac{\textit{Adjacent side}}{\textit{Opposite side}} = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{\textit{Hypotenuse}}{\textit{Opposite side}} = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{\textit{Hypotenuse}}{\textit{Adjacent side}} = \frac{1}{\cos\theta}$$

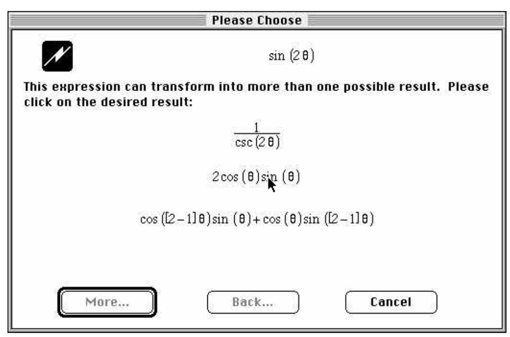
This relationship leads to several more algebraic identities which you can use to transform and simplify trigonometric equations. *MathView* has the basic trigonometric functions defined internally, but leaves the user flexibility by defining most trigonometric relationships as transformation rules.

The distributed *Trig Functions* notebook contains many of these, and allows you to define as many others as you want. The example below uses a rule found in the “New Notebook”. In this case, as in many others, you have more than one choice.



- Input and select the expression below. Choose **Transform** from the **Manipulate** menu, and click on one of the choices in the choice box.

Expression selected and a **Transform** performed



To select one of the choices, just click on top of your choice.

Result

Polar vs. Rectangular Coordinates

You use two PreDefined functions, **FromPolar(x)** and **ToPolar(x)**, to translate polar into rectangular coordinates and rectangular into polar coordinates, respectively. *MathView* does not define these functions in a “New Notebook”, so the first time you try to use one, the program will ask you to define it.

MathView uses the function **FromPolar** for polar plots, and will ask that you PreDefine it the first time you generate that type of plot.

- Input the following two Assumptions and calculate. Click on the PreDefined default button when asked to declare the functions. This will make them available for the present operation and all future manipulations in the notebook.

□ FromPolar(2, 2)
△ FromPolar(2, 2) = (−0.83229, 1.8186) *Calculate*
□ ToPolar(−0.83229, 1.8186)
△ ToPolar(−0.83229, 1.8186) = (2, 2) *Calculate*

In similar fashion, three-vector conversion between rectangular and cylindrical coordinates (r, θ, z), and rectangular and spherical coordinates (r, θ, ϕ), are available through the use of the **Cylindrical** and **Spherical** PreDefined functions.

Dynamic Circle

The next example uses the functions just discussed and demonstrates the tremendous flexibility of *MathView*.

The goal of this example is to create a Graph Theory which plots a circle with its center at the origin. In addition, given a radius and an angle in degrees, have *MathView* plot a line which represents the given ray. By changing the radius and angle, the circle and ray will re-draw with the new dimensions.

- In two Props enter the magnitude, $m = 1$, and the angle, $n = 45$ (in degrees), of the ray.
- Given a point on the circle ***p*** in polar coordinates with the angle in degrees, use the two functions discussed earlier to define the point. Define ***p*** as a **M-Linear Operator** (a matrix definition).

⦿ p = FromPolar(m, DegToRad[n])

Since the angle is given in degrees, we need **DegToRad** (***n***) to transform the angle into radians, a requirement of the **FromPolar** function. The resulting point ***p*** is now in rectangular coordinates.

- Select the RHS and **Calculate**.

⦿ m = 1
⦿ n = 45 ← Angle in degrees
⦿ p = FromPolar(m, DegToRad[n]) ← Select RHS and **Calculate**
△ p = (0.70711, 0.70711) *Calculate*

- Next you use a subscripted index to extract each coordinate from this last conclusion (you enter subscripts by typing the variable then an underline followed by the subscript).

□ $p_{(1, 1)}$ ← Select and **Calculate**
△ $p_{(1, 1)}$ = 0.70711 *Calculate*
□ $p_{(1, 2)}$ ← Select and **Calculate**
△ $p_{(1, 2)}$ = 0.70711 *Calculate*

These two numbers define the point in ***x-y*** coordinates. You can now use them to define the ray by entering them into an equation for a line (Point Slope form).

- Input the equation for the line in a new Prop (see below); **Substitute** the points into the equation and **Isolate** ***y***.

$$y - p_{(1,2)} = \frac{p_{(1,2)}}{p_{(1,1)}}\left(x - p_{[1,1]}\right)$$

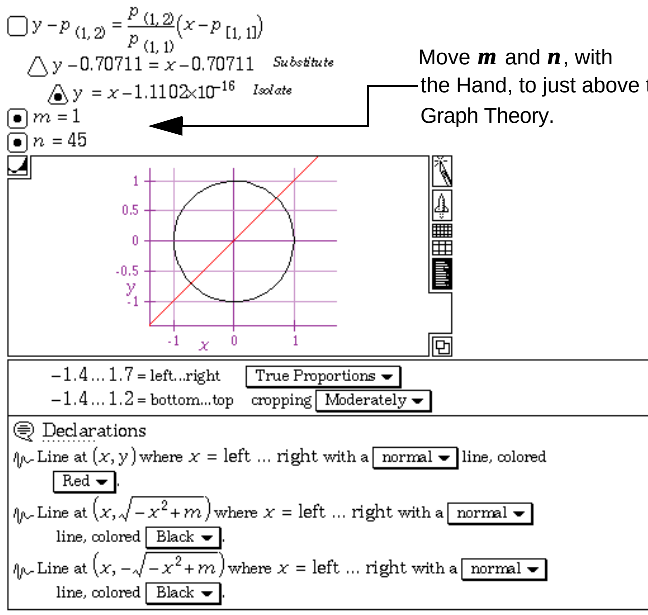
$$y - 0.70711 = x - 0.70711 \quad \textit{Substitute}$$

$$y = x - 1.1102\times10^{-16} \quad \textit{Isolate}$$

line equation

- Next, you create the Graph Theory which will plot this line along with the associated circle. Select the Prop containing the line equation above and choose **Linear** under the **Graph** menu.
- Add two line plots and define the first as the top half and the second as the bottom half of a circle. Make the circle dynamic (meaning that the plot re-draws the circle when you change the value of ***m***) by changing the **1** in each equation to an ***m***.
- With the Knife or the Rocket Ship, adjust the Viewport to accommodate the plot of the circle, and set the details to **True Proportions** . You can now change the graph by giving new values to either ***m*** or ***n***.

You can generate this same graph by using an implicit plot (under **Other ▶ f(x,y) = g(x,y) Implicit**). Plot the circle $x^2 + y^2 = m$ and add the line plot by selecting the Prop with the line equation and choosing **Add Line Plot**. You may have to increase the resolution of the plot to make a



Move ***m*** and ***n***, with the Hand, to just above the Graph Theory.

Scatter Plots & Data Analysis

• Entering and plotting Matrix values
• Log and Scatter plots
• Matrix indexing

The curve-fitting example which follows demonstrates how you enter and plot unequally spaced data points using *MathView's* matrix operator and Scatter-plot Graph Theories. Using the same data, you will expand the scope of the problem, giving you experience generating a Log-Log plot.

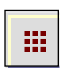
MathView's Table feature allows only equally spaced numbers for the ***x*** values (domain). You must create a matrix to plot a set of data points which have unequally spaced input values. In the next example you analyze a set of data by plotting the points in a scatter plot and then you determine an equation which fits that data.

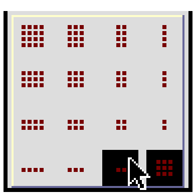
See page 88 for further instructions on how to enter matrices.

- Inside a Case Theory, input the data in the **D=** Prop below by creating an equation with the RHS in the form of a matrix. You can enter it by typing

```
D = 1 , .2893 ; 1.8 , 5.6592 ; 2.6 , 36.363 ; ...
```

Notice how *MathView* automatically generates the parentheses and creates a new row after each semi-colon.



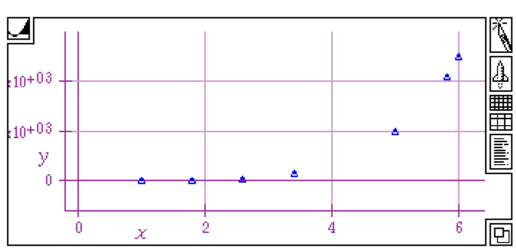


- The second method is to use the matrix palette button. Type ***D*** = and select a two column matrix from the pop-up menu. Type the first number, a comma, and then a semi-colon. You can alternatively select a 4 row by 2 column matrix from the same pop-up palette. You would then tab through, typing the members until you get to the end of the fourth row. Type a semi-colon to add additional rows. The Prop should look like the following when complete.

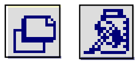
$$D = \begin{pmatrix} 1 & 0.2893 \\ 1.8 & 5.6592 \\ 2.6 & 36.363 \\ 3.4 & 141.27 \\ 5 & 993.96 \\ 5.8 & 2106 \\ 6 & 2500 \end{pmatrix}$$

- **Clarify** the notebook and declare ***D*** a **M-Linear Operator** (**M** means matrix).
- Select the equation, by clicking on the equal sign, and generate a scatter plot by choosing **Scatter ▶ Linear** under the **Graph** menu. Adjust the Graph Theory to look similar to the following.

Case Theory #1

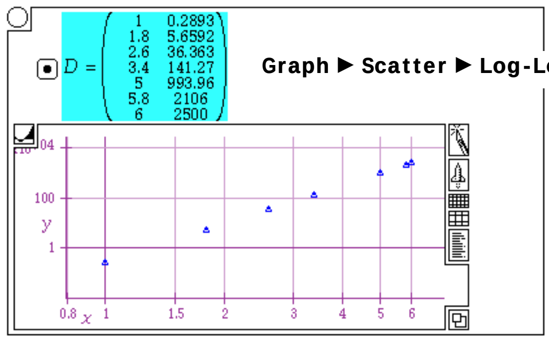


The plot looks like it has the shape of an exponential curve. To verify that the curve is exponential, you need to determine whether a linear relationship exists between the log of the ***x***s and the log of the ***y***s. You can verify this relationship in one of two ways.



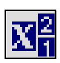
- First is to plot the data in a log-log plot by selecting the equation and choosing **`Log-Log`** under **`Scatter`** in the **`Graph`** menu. Since you have already generated one plot in the notebook, copy the equation and paste it into a new Prop. Select the Prop and create a Case Theory to keep this Theory separate from the other one. Select the equation again and generate the Log-Log plot.

Case Theory #2



`Graph ▶ Scatter ▶ Log-Log`

The second method requires you to explicitly take the logarithms of the data.

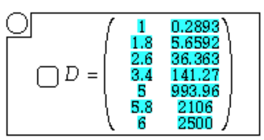


- Copy the matrix equation again and Paste into a third Case Theory. Select the RHS by dragging the mouse through the data and perform a **`Select In`** (click on the palette icon or choose **`Edit ▶ Select In`**).
- Apply the Natural Log function by clicking on the image of **ln(x)** on the functions palette.

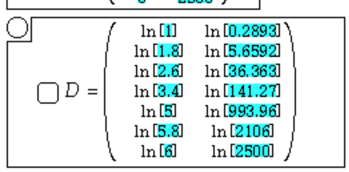
- Select the RHS again and hold down the [option] and [⌘] keys (Mac) or the [alt] and [Ctrl] keys (Win), while you **`Calculate`**. This performs a calculation in place. Select the equation and generate a new linear scatter plot.

Below, and on the next page, are the steps and the resulting plot.

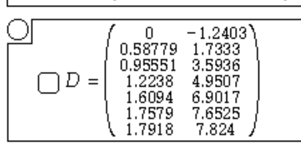
Case Theory #3



After you have selected the RHS and have performed a **`Select In`**.

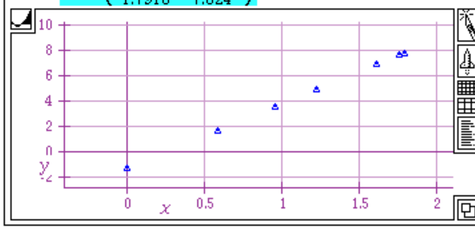


After you click on **`ln(x)`**.



After **`Calculated`** in place.

$$D = \begin{pmatrix} 0 & -1.2403 \\ 0.58779 & 1.7333 \\ 0.95551 & 3.5936 \\ 1.2238 & 4.9507 \\ 1.6094 & 6.9017 \\ 1.7579 & 7.6525 \\ 1.7918 & 7.824 \end{pmatrix}$$



After Scatter Plot Generated.

Case Theory #3

You can see that an exponential model will fit the data because both of the plots above show a linear relationship. To verify, you must determine the equation for the linear fit.

- Create the following equation for the slope of that line inside the third Case Theory.

$$m = \frac{D_{(7,2)} - D_{(1,2)}}{D_{(7,1)} - D_{(1,1)}}$$

$$m = 5.0589 \quad \textit{Calculate}$$

Slope of the line.

The reason you declare ***D*** an **M-Linear Operator** now becomes apparent. To determine the slope, you must extract the values of two points. You extract these values by indexing the matrix as shown above. Use the first and last points for this operation.

- Input an equation for a line using one of the points (use the 7th point) to determine ***b*** (***y*** intercept). **Substitute *m*** into this equation and **Isolate *b***.

$$7.824 = m(1.7918) + b$$

$$7.824 = b + 9.0643 \quad \textit{Substitute}$$

$$b = -1.2403 \quad \textit{Isolate}$$

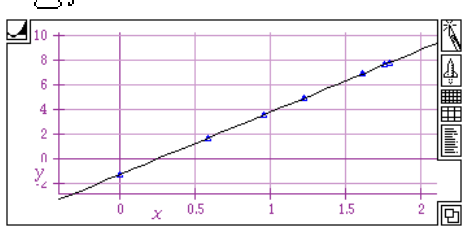
- Create another equation for a line, in general form, and substitute ***m*** and ***b*** into it. **Isolate *y***.

- Select the Graph Theory and choose **Add Line Plot** from the **Graph ▶ Additional** menu.

Case Theory #3

$$y = mx + b$$

$$y = 5.0589x - 1.2403 \quad \textit{Substitute}$$



This plot verifies that the **ln** model is linear.

You determine the equation that will fit the original data by using the values of ***m*** and ***b*** to write ***y*** as an exponential function of ***x***.

- In Case Theory #1, re-input the equations for m and b directly beneath the existing graph.
- Input the equation $\ln(y) = m\ln(x) + b$.
- Substitute m and b into this equation.
- **Isolate** ***y***.
- **Collect** the RHS with **Auto Simplify** OFF and **Calculate**.

$m = 5.0589$
$b = -1.2403$ — Re-input ***m*** and ***b*** just below the 1st Plot.

$\ln(y) = m\ln(x) + b$

$\ln(y) = 5.0589\ln(x) - 1.2403$ *Substitute*

$y = e^{5.0589\ln(x) - 1.2403}$ *Isolate* ← **Collect** with **Auto Simplify** OFF.

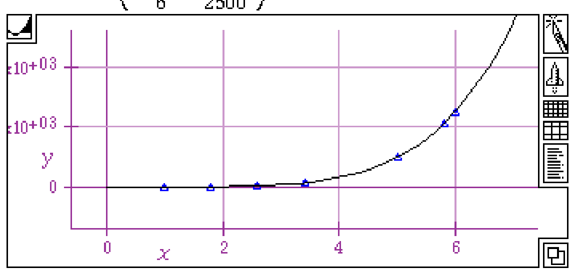
$y = e^{5.0589\ln(x)} e^{-1.2403}$ *Collect*

$y = 2.7183^{5.0589\ln(x)} \cdot 0.2893$ *Calculate*

- Add a line plot to the graph by selecting the last ***y*** Prop and choosing **Add Line Plot** under **Graph ▶ Additional** menu. The graph will look like the following with the line fitting the data.

Case Theory #1

$$D = \begin{pmatrix} 1 & 0.2893 \\ 1.8 & 5.6592 \\ 2.6 & 36.363 \\ 3.4 & 141.27 \\ 5 & 993.96 \\ 5.8 & 2106 \\ 6 & 2500 \end{pmatrix}$$



$m = 5.0589$

$b = -1.2403$

$\ln(y) = m\ln(x) + b$

$\ln(y) = 5.0589\ln(x) - 1.2403$ *Substitute*

$y = e^{5.0589\ln(x) - 1.2403}$ *Isolate*

$y = e^{5.0589\ln(x)} e^{-1.2403}$ *Collect*

$y = 2.7183^{5.0589\ln(x)} \cdot 0.2893$ *Calculate*

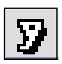
Function Algebra & Composite Functions

- Using Wildcard Variables to manipulate functions
- Turning off Auto Simplify to show steps

You can automate the combining of functions by using functions entered in Wildcard form. Once you have entered them in Wildcard form, you perform simple mathematical manipulations with them.

Sums and Differences

Recall, to enter a Wildcard variable, type a ? just prior to typing the letter (***x*** in this case), or choose x from the Pop-up menu.



- Enter the following two functions in two separate Props using Wildcard variables for the independent variable.

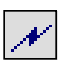
$$f(x) = \frac{\sqrt{x} + \sin(x)}{3x}$$

$$g(x) = \frac{1}{x}$$

- In a third and forth Prop, enter an addition and subtraction of the functions. This time do not use Wildcard variables.

$$f(x)+g(x)$$

$$f(x)-g(x)$$



- Select both functions by clicking on their equal signs while holding down the shift key. Substitute into each of the Props above. Turn **Auto Simplify** off so you can see the steps that *MathView* takes to solve these operations. Select the RHS of the results and **Expand**.

$$f(x) = \frac{\sqrt{x} + \sin(x)}{3x}$$

$$g(x) = \frac{1}{x}$$

Select these two equations and **Substitute** into #1 and #2 below.

#1 $f(x)+g(x)$

$$f(x)+g(x) = \frac{1}{x} + \frac{1}{3}\frac{\sin(x)+\sqrt{x}}{x} \quad \text{Substitute}$$

$$f(x)+g(x) = \frac{1}{x} + \frac{1}{3}\frac{1}{\sqrt{x}} + \frac{1}{3}\frac{\sin(x)}{x} \quad \text{Expand}$$

#2 $f(x)-g(x)$

$$f(x)-g(x) = -\frac{1}{x} + \frac{1}{3}\frac{\sin(x)+\sqrt{x}}{x} \quad \text{Substitute}$$

$$f(x)-g(x) = -\frac{1}{x} + \frac{1}{3}\frac{1}{\sqrt{x}} + \frac{1}{3}\frac{\sin(x)}{x} \quad \text{Expand}$$

Multiplication and Division

You use the same method to multiply and divide functions. Use the same two functions for this example.

- Enter two new Props defining the multiplication and the division of the functions.
- Substitute the functions into the second and third Props shown below.

$$f(\square) = \frac{\sqrt{\square} + \sin(\square)}{3\square}$$

$$g(\square) = \frac{1}{\square}$$

$$f(x)g(x)$$

$$\triangle\ f(x)g(x) = \frac{1}{3}\frac{\sin(x) + \sqrt{x}}{x^2} \quad \textit{Substitute}$$

$$\triangle\ f(x)g(x) = \frac{1}{3}\frac{1}{x^{\frac{3}{2}}} + \frac{1}{3}\frac{\sin(x)}{x^2} \quad \textit{Expand}$$

$$\frac{f(x)}{g(x)}$$

$$\triangle\ \frac{f(x)}{g(x)} = \frac{1}{3}(\sin[x] + \sqrt{x}) \quad \textit{Substitute}$$

$$\triangle\ \frac{f(x)}{g(x)} = \frac{1}{3}\sin(x) + \frac{1}{3}\sqrt{x} \quad \textit{Expand}$$

Composite Functions

Composite functions play an important role in higher mathematics. Of particular interest is their use in differential calculus when studying the chain rule and in integral calculus when studying the substitution rule.

Not only can *MathView* help you solve these problems, it can help you visualize the results graphically. The example below uses two simple functions to show how; by looking at the graph of the two functions, and the composite, you can determine the domain of the resulting function.

- Input the two functions below along with the composite $g(f(x))$.

$$f(\square) = \square^2 - 2$$

$$g(\square) = \sqrt{\square}$$

$$g(f[x])$$

$$\triangle\ g(f[x]) = \sqrt{f(x)} \quad \textit{Substitute}$$

$$\triangle\ g(f[x]) = \sqrt{x^2 - 2} \quad \textit{Substitute}$$

You must first substitute $g(x)$ then $f(x)$ into that result to obtain the final result.

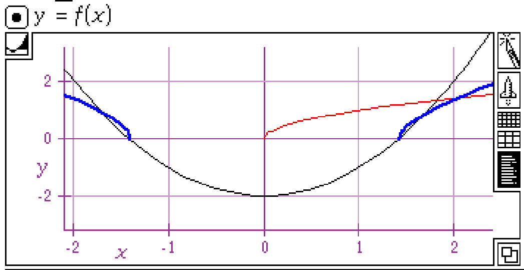
- Create three equations defining the two initial functions and the composite function. Use different dependent variables so you can plot all three in the same Graph Theory. You can use prime notation (see below), you can index the dependent variables, or you can use entirely different User Defined variables. You merely need to distinguish the three equations for the Graph Theory to work.

$\square\ y = f(x)$
$\square\ y' = g(x)$
$\square\ y'' = g(f[x])$

- Select the first Prop above and generate a linear Graph Theory by choosing **y = f(x) ▶ Linear** under the **Graph** menu.
- In order, select each of the other equations by clicking somewhere inside its Prop, and choose **Add Line Plot** under the **Graph ▶ Additional** menu. Declare each new variable User Defined.

After some adjustment, the Graph Theory will look something like the following (the composite function is the heavy line).

Declarations
$f(x) = x^2 - 2$
$g(x) = \sqrt{x}$
$g(f[x])$
△ $g(f[x]) = g(x^2 - 2)$ *Substitute*
△ $g(f[x]) = \sqrt{x^2 - 2}$ *Substitute*
$y = f(x)$



$-2.1 \ldots 2.4$ = left...right Stretch to Fit
$-3.2 \ldots 3.2$ = bottom...top cropping Moderately

Declarations
Line at (x, y) where x = left ... right with a normal line, colored Black.
Line at (x, y') where x = left ... right with a normal line, colored Red.
Line at (x, y'') where x = left ... right with a heavy line, colored Blue.

$y' = g(x)$
$y'' = g(f[x])$

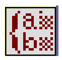
The plot shows that the domain of g is all positive numbers, but the domain of the composite is $g(f(x)) = (-\infty, -\sqrt{2}] \quad [\sqrt{2}, \infty)$.

Piecewise-Defined Functions

- Using the Conditional Op to plot piecewise functions
- Re-defining line details

You enter piecewise-defined functions by using the Conditional Op.

- Input the following function by typing **`y = Conditional(`**To input the relations, start by creating a two column matrix (click on the palette image for this). Tab through the matrix to input the expressions below.



You can also enter the Conditional Op by clicking on the palette image. This inputs a two-relation Op which you can then tab through to place your inputs.

- To add a third row, type a semi-colon at the end of the second row.

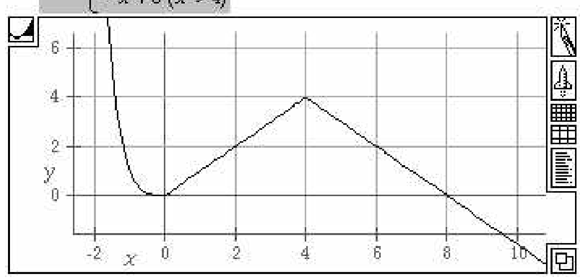
$$y = \begin{cases} ? & (? < 0) \\ ? & (? \geq 0) \end{cases} \qquad y = \begin{cases} x^4 & (x \leq 0) \\ |x| & (x \leq 4) \\ -x+8 & (x > 4) \end{cases}$$

- Select the equation by clicking on the equal sign and generate a Linear Graph Theory.

The relational operators default to ones different from the stated problem. To change, select each by clicking once on the Op and press delete. Re-enter by using the palette image inside the **a+b** pop-up menu, or by typing in the correct one.

MathView interprets a conditional from the top down. If you change the order of the entries, you may get unexpected results

$$y = \begin{cases} x^4 & (x \leq 0) \\ |x| & (x \leq 4) \\ -x+8 & (x > 4) \end{cases}$$



$-2.6 \ldots 10.8$ = left...right Stretch to Fit
$-1.6 \ldots 6.6$ = bottom...top cropping Moderately

Use these bounds.

A second method works as well, but takes somewhat longer to input.

- Delete the last example and input the first conditional equation $y = x^4$ in its own Prop. Plot it by generating a linear graph. Add two line plots and redefine their details as follows. *MathView* generates the same plot.

Declarations

Line at (x, y) where x = left ... 0 with a normal line, colored Black.

Line at $(x, |x|)$ where $x = 0 \ldots 4$ with a normal line, colored Red.

Line at $(x, -x+8)$ where $x = 4 \ldots$ right with a normal line, colored Blue.

Add these two lines

Functions of Two Variables

- 3-D Plots
- Contour Plots
- Adding surface plots to an existing 3-D graph
- Adding a 2-D plane to a 3-D Graph Theory

With the exception of time, we live in a three-dimensional world. You can use functions of two variables to mathematically describe this world and *MathView* has the tools you need. The process of describing these functions and manipulating them is very similar to the two-dimensional functions introduced earlier.

You can enter functions of two variables in explicit form or as User Defined functions. Use the following function for the next example.

$$z = \frac{xy(x^2 - y^2)}{x^2 + y^2}$$

- Enter the equation in explicit form and as a User Defined function in separate Props. In a third Prop, enter $z = f(x, y)$.

Target Prop #1 $\quad$ $z = \frac{xy(x^2-y^2)}{x^2+y^2}$

$f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$

Target Prop #2 $\quad$ $z = f(x,y)$

$x = 3$

$y = 2$

- Add two Props defining values for ***x*** and ***y*** (see above). Substitute ***x*** and ***y*** into each of the two target Props by clicking on their equal signs while holding down the Shift key. With the Hand move to the target Props #1 and #2.

The first substitution generates ***z*** in fractional form. The second leaves ***z*** as a function.

- **Calculate** the RHS of each result for a decimal answer. In #2, you could also **Substitute** the function (Wildcard form) into the $z = f(3, 2)$ Prop. This method gives the same intermediate fraction as in #1. **Calculate** that result for the decimal answer.

#1

$z = \frac{xy(x^2-y^2)}{x^2+y^2}$

$z = \frac{30}{13}$ *Substitute*

$z = 2.3077$ *Calculate*

#2

$f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$

$z = f(x,y)$

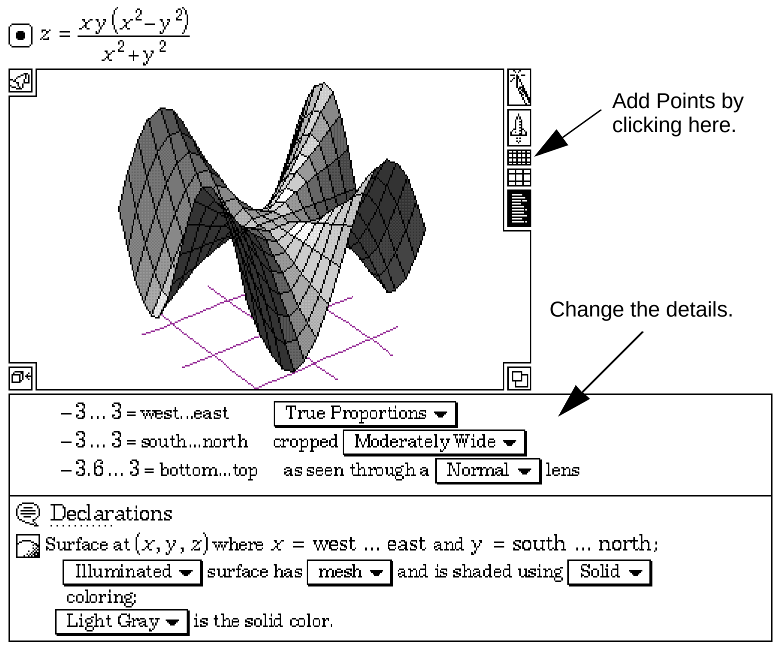
$z = f(3,2)$ *Substitute*

$z = 2.3077$ *Calculate*

Color and Illuminated 3-D Plots

- Plot the function by selecting either equation and choosing **z = f(x,y) ▶ Color 3D** under the **Graph** menu (the graphic below uses the first equation and has an illuminated surface). Rotate the plot to an orientation similar to the Graph Theory below or merely copy the details. Increase the number of points by clicking on the mesh icon. Open the Declarations and remove the axes by selecting their leading icons and press delete).

The difference between **Illum. 3D** and **Color 3D** graphs is merely the color and lighting. **Illum. 3D** graphs will appear to be illuminated by a light source over your left shoulder.

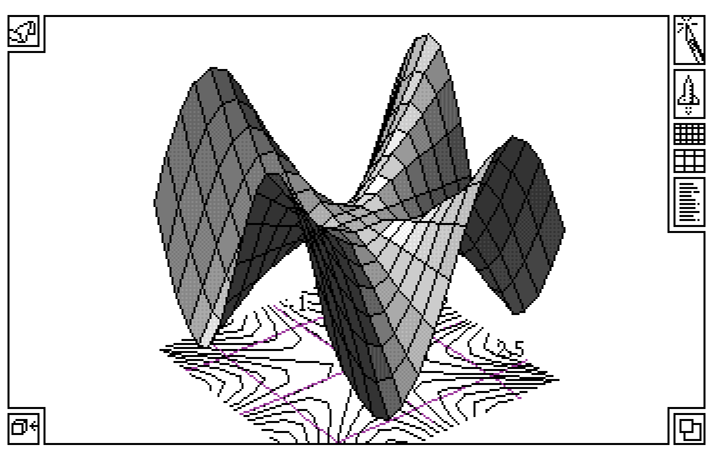


Examples of graphs which plot more than one variable in two dimensions are topographical and weather maps. The Graph Theory below shows an interesting variation on the last example by generating the contours associated with the 3-D graph.

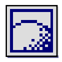
To generate a contour plot *by itself*, select the **z** equation and choose **Graph ▶ z=f(x,y) ▶ Contour 2D**.

- Select the function again and choose **Add Contour Plot** under the **Graph ▶ Additional** sub-menu. The following Graph Theory displays the results.

You may want to remove the grid-lines in the contour plot so you can see the contours better. Open the graph details, select the gridline detail inside the Declarations, and press delete.

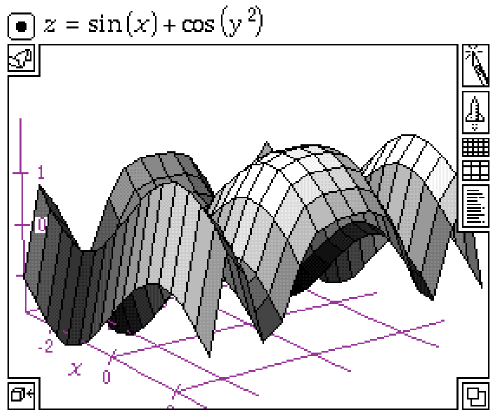


Traces and Partial Derivative Functions



You can plot a surface of intersection by adding a second plot and describing it as a plane. This method can help you visualize partial derivatives.

- In a new notebook, enter the following equation and generate a 3-D graph. The choice of colors and detail is up to you.

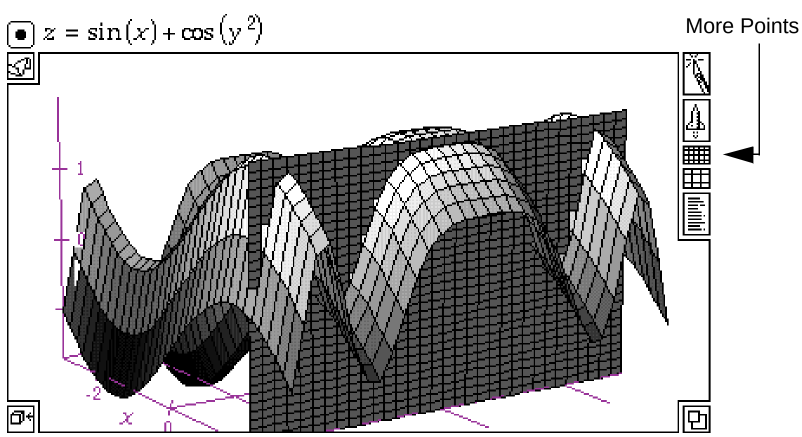


Orentate the graph so it looks like this.

- To create the plane, select **with blank slots** under the **Graph ▶ Additional** menu. This selection will place a check mark to the left of **with blank slots**.
- Add a Surface Plot by choosing **Graph ▶ Additional ▶ Add Surface Plot**. Since you have **with blank slots** turned on, the added surface plot will have its parameters blank.
- Change **(?, ?, ?)** to **(2, *y*, *z*)** and change the two ranges to **y=south...north** and **z=bottom...top** . The number **2** was choosen for the location of the plane, but you can choose any number as long as it is within the range of the Viewport.

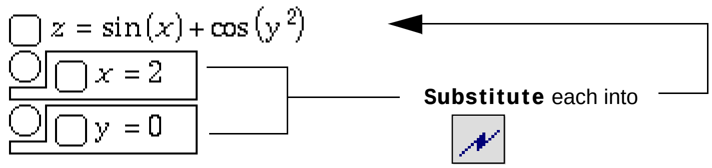
Surface at (2, y, z) where y = south ... north and z = bottom ... top ; Opaque surface has mesh and is shaded using Solid coloring; Dark Gray is the solid color.

The resulting Graph Theory looks like the following after you add more points by clicking on the mesh icon.

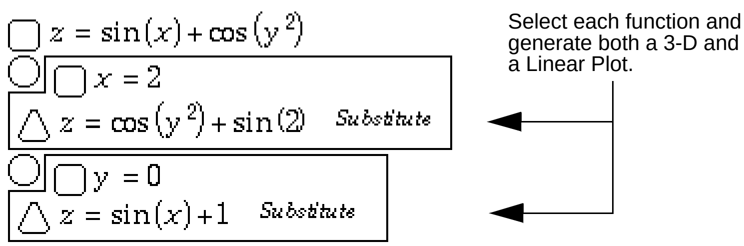


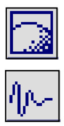
The curve of intersection of the two surfaces graphically shows the partial derivative function with respect to ***x***. You can also graphically study the partial derivative function with respect to ***y***.

- Generate two new equations defining the points, $x = 2$ and $y = 0$. Surround each in a Case Theory.



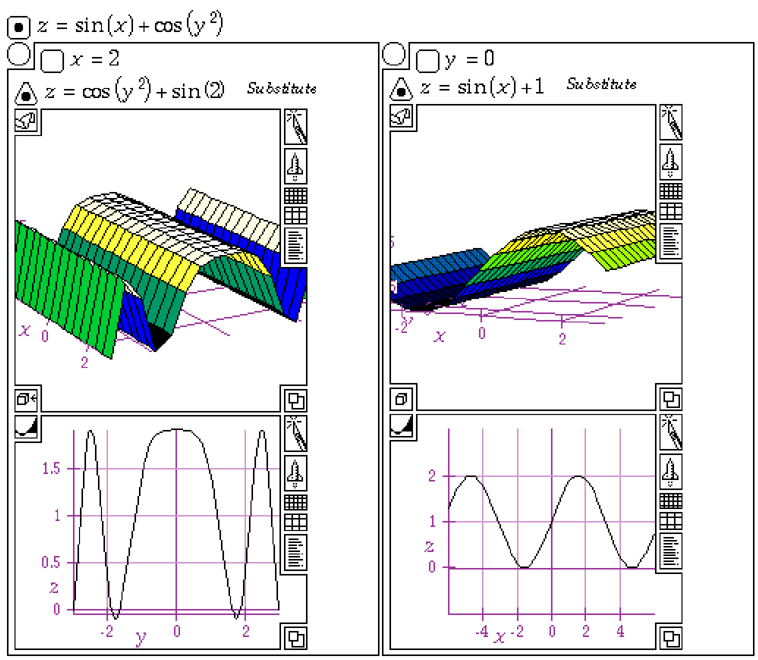
- Substitute the ***x*** and ***y*** Props into the ***z*** function (see above) to generate the derived conclusions.





- Select each function and generate a 3-D plot. Select the functions again and generate a Linear plot. Move them side by side, to fit on the page, like this.

Make sure you choose the correct variables for your graphs when the dialog appears.



Parametrized Curves and Polar Coordinates

- Plotting and animating Parametric plots
- Plotting and animating Space curves
- Plotting and animating Polar plots

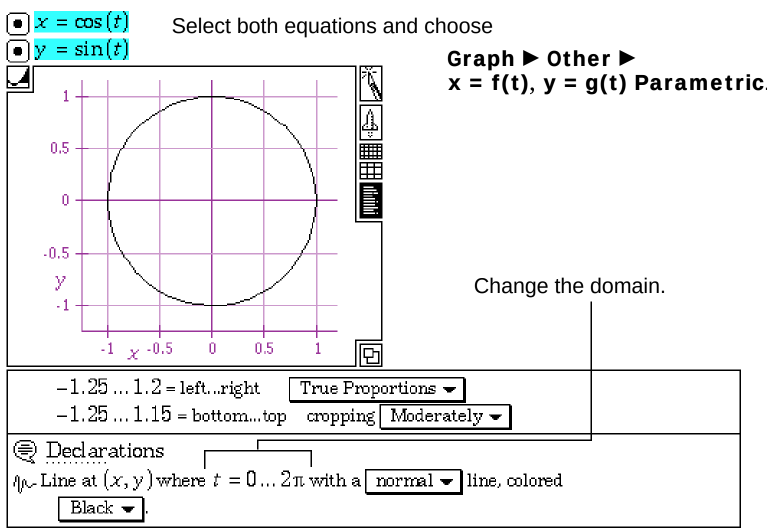
You can describe and visualize the motion of particles using parametric plots. Using polar coordinates for some relations can add simplicity and insight which is difficult using cartesian coordinates. *MathView* allows you to do both, using two powerful graphing facilities.

Parametric Plots

You can graph circles in *MathView* by using a Linear Graph Theory, **Graph ▶ y = f(x) ▶ Linear**, where you plot the two equations defining the halves of the given circle in the same Theory. You can also use the Implicit Graph Theory, **Graph ▶ Other ▶ f(x,y)** = **g(x,y) Implicit**, where you plot the circle using one equation.

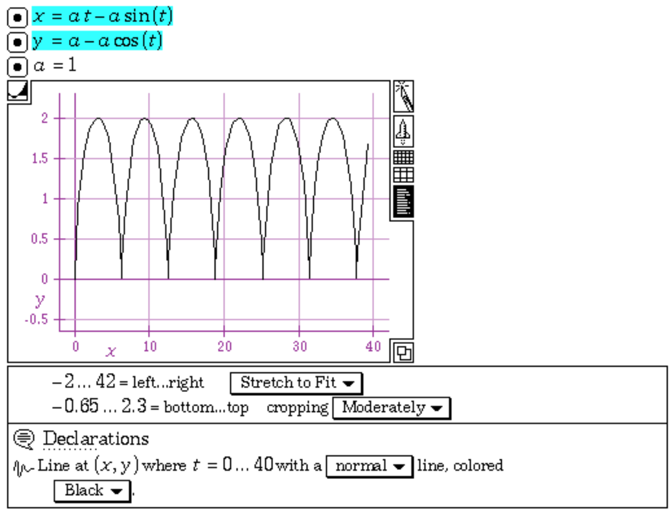
When you want to describe the location of a particle on a circle given the central angle, a better method to use is the Parametric Graph Theory, **Graph ▶ Other ▶ x = f(t)**, **y = g(t) Parametric**. The example below plots a circle with a radius of one using this method.

- Enter the two equations below. Select both, by clicking on their equal signs while holding down the shift key, and choose the Parametric Graph Theory under the **Graph ▶ Other** menu. Declare t as a variable and, to better show the plot, change the domain of t from $t = -3$ 3 to $t = 0$ 2π and the Viewport to **True Proportions** .



You can obtain additional insight by animating a Parametric plot.

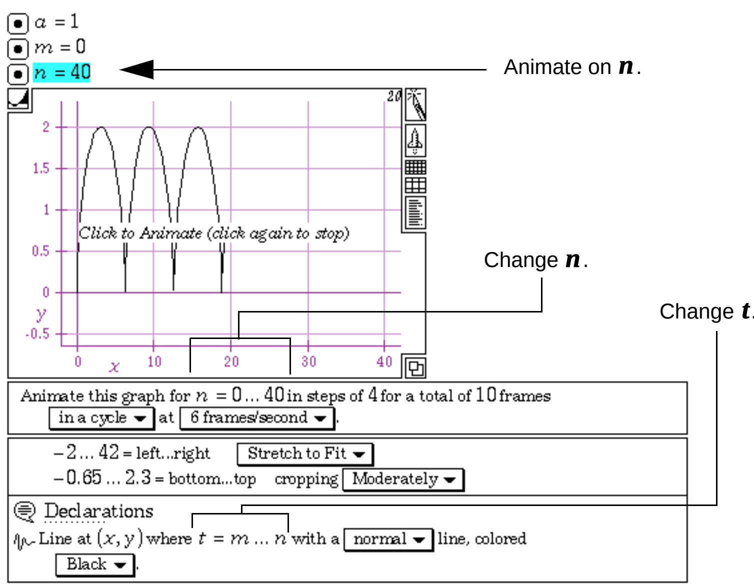
- In a new notebook, or inside a Case Theory within the notebook you just used, enter the equations for a cycloid. Select both equations and plot using the same method used in the prior example. The parameter ***a*** defines the location of the center of the circle above the ***x***-axis. Enter ***a*** equal to **1** in a third Prop. Change the domain to $t = 0 \quad 40$.



By keeping the proportions **Stretch to Fit** as you animate, you can observe a very interesting animation.

Declare all variables User Defined.

- Add two equations $m = 0$ and $n = 40$ right beneath the $a = 1$ Prop. Open the graph details and change the domain to $t = m \quad n$. Choose the $n = 40$ equation and animate (**Graph ▶ Animation ▶ Start**). The default animation range is short, so change it to $n = 0 \quad 40$ after you have stopped the animation.
- Restart the animation.



Space Curves

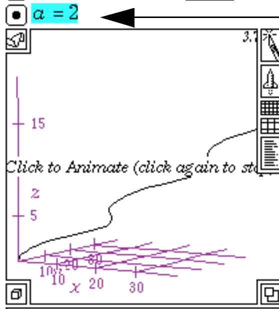
You can expand the previous example to show how *MathView* plots Space Curves. You do this by adding a third equation which defines the third dimension.

- In a new Prop inside a Case Theory or in a new notebook, input the same equations found in the last example. Add the equation $z = t$. Select all three equations and choose **Graph ▶ Other ▶ x = f(t), y = g(t), z = h(t), Space Curve**. Accept the default graph variables or change to match the equation variables and click **OK**. Change the details to match the ones below.
- An interesting addition to this graph is to animate on the ***a*** variable. Select the $a = 2$ Prop and **Animate**.

$x = a\,t - a\sin(t)$
$y = a - a\cos(t)$
$z = t$
$a = 2$

Select these 3 equations to Plot.

Animate on ***a***.



Animate this graph for $a = 0\ldots15$ in steps of $\frac{3}{4}$ for a total of 20 frames in a cycle at 4 frames/second.

0 … 40 = west…east Stretch to Fit
0 … 40 = south…north cropped Moderately
0 … 20 = bottom…top as seen through a Normal lens

Declarations
Line at (x, y, z) where $t = 0\ldots20$ with a normal line, colored Black.

Polar Plots

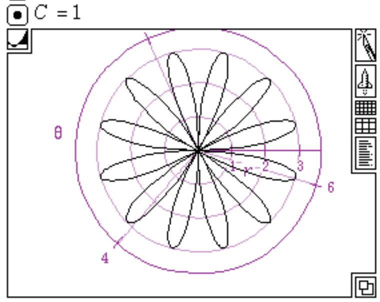
MathView automatically declares the **ToPolar** function when you ask it to graph Polar Plots. The following example again shows the interesting and powerful animation feature of *MathView.*

- Enter the equations below and change the values of ***A***, ***B***, and ***C*** (declare each variable User Defined).

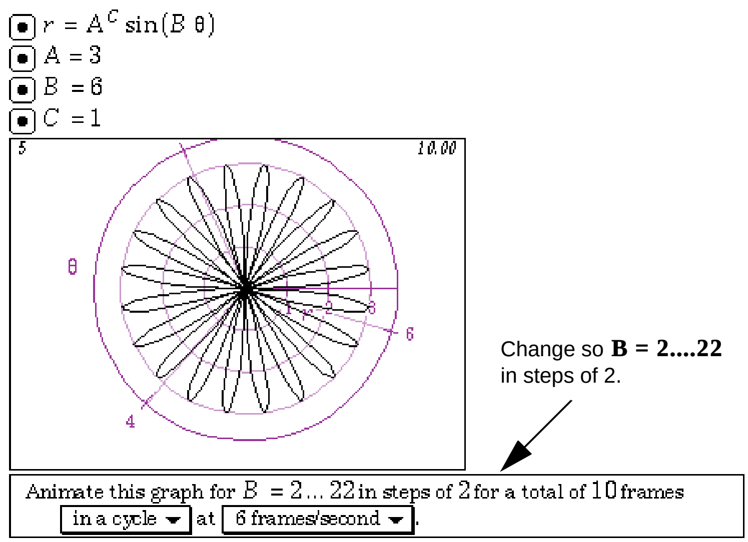
For this plot, define the ***x***-axis as *r* and the ***y***-axis as θ.

$r = A^{C}\sin(B\,\theta)$
$A = 3$
$B = 6$
$C = 1$

Declare θ a User Defined variable. Select and choose **Graph ▶ y = f(x) ▶ Polar**.



- Select the $B = 6$ Prop and **Animate**. Change the animation details to generate even increments of the variable from **2** to **22**. The screen shot below caught the animation at $B = 10$.



Saving an Animation (Mac only)

If you are using the Macintosh version of *MathView*, you can save any animation you create by choosing **Save...** under **Animation** in the **Graph** menu.

- Click on the animation to be saved and choose **Graph** ▶ **Animation** ▶ **Save...**. The program will go through the animation one time, saving each frame to a file.
- When all frames have been saved, a dialog box will open where you can name the animation and save it to a file of your choice. Name the animation Polar Animation and click on the **Save** button or press the Return/Enter key.

The file placed on your disk is a QuickTime file which you can then use in various programs compatible with QuickTime. If you do not have QuickTime installed on your machine, saved animations use the PICT format. The images saved are bitmapped images with the same “depth” (number of bits per pixel) as your computer screen. If you are using more than one monitor, the screen with the deepest pixel depth determines the depth of the saved images.

Solving Equations Graphically

- Solving simultaneous equations using a Linear Graph Theory
- Solving simultaneous equations using Implicit Plots

MathView provides you several ways to solve equations. In the section on Matrices (page 88) you solved a simple system of equations using algebraic manipulation, and you solved a more complicated system of equations using matrices. In this section you will learn how to use *MathView's* Graph Theories to solve equations.

Given the following two equations, solve for ***x*** and ***y***.

Remember that if you have two expressions equivalent to y, then the two expressions are equal to each other.

- In two separate Props enter the following two equations.

$$x + \sin(x) + y = 3$$
$$y = \sin(x) + 2$$

The first method to solve two equations graphically is to create a single equation by substitution.

- Substitute the second equation into the first.

$$x + \sin(x) + y = 3$$
$$2\sin(x) + x + 2 = 3 \quad \textit{Substitute}$$

- Move the LHS over to the RHS.

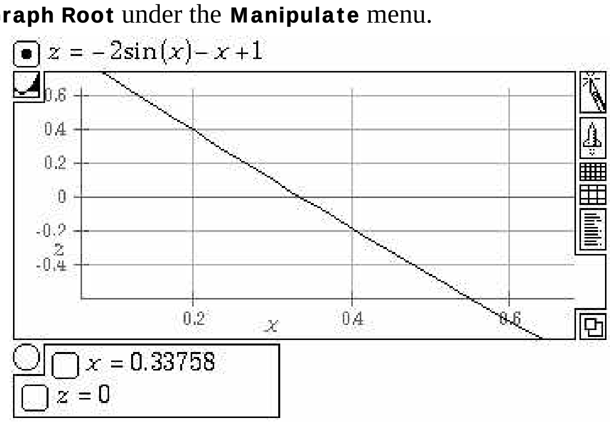
$$2\sin(x) + x + 2 = 3 \quad \textit{Substitute}$$
$$0 = -2\sin(x) - x + 1 \quad \textit{Move Over}$$

Assign the value z to this new equation so that *MathView* knows what to graph.

- Select the zero in this new Conclusion and type a **z**.

$$z = -2\sin(x) - x + 1$$

- Plot this function in a Linear Graph Theory, zoom in on the root and select **Find Graph Root** under the **Manipulate** menu.



All you need do now is **Substitute** the ***x*** Prop into one of the original equations and solve for ***y***.

$x = 0.33758$

$z = 0$

$y = \sin(0.33758) + 2$ *Substitute*

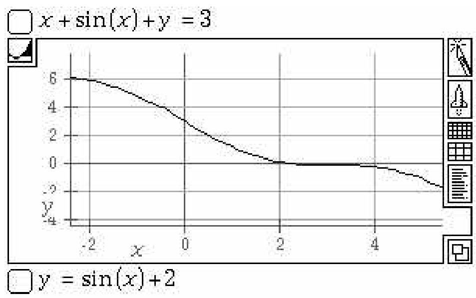
$y = 2.3312$ *Calculate*

A second method uses *MathView's* Implicit Plot facility. This facility is specifically designed for two equations and two variables. In fact, you must use this method when expressing an equation explicitly is impossible.

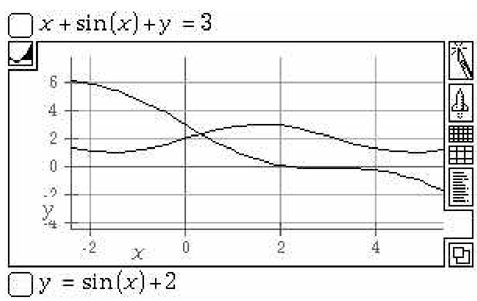
An explicit equation has only a single variable on the LHS:

$y = \sin(x) + 2$

- Enter both equations, select the first, and generate an Implicit Plot by choosing **f(x,y)=g(x,y) Implicit** under the **Graph ▶ Other** menu. Increase the resolution and adjust the Viewport to look something like the following.



- Select the second equation and choose **Add Implicit Plot** from the **Graph ▶ Additional** menu.



- Zoom in on the plot where the curves cross and choose **Find Graph Root** under the **Manipulate** menu to obtain a numerical solution.