

# Calculus&LiveMath

## Growth

(Differential Calculus)

### Detailed Syllabus

#### Mathematics

- Line functions and polynomials.
- Interpolation of data.
- Compromise lines through data.
- Dominant terms in the global scale.
- How to write exponential and logarithm functions in terms of the natural base  $e$ .
- While line functions post a constant growth rate, exponential functions post a constant percentage growth rate.
- How to construct a function with a prescribed percentage growth rate.
- The instantaneous growth rate  $f'[x]$  as the limiting case of the average growth rates  $(f[x+h] - f[x])/h$ .
- Calculation of  $f'[x]$  for functions  $f[x]$  like  $x^k$ ,  $\sin[x]$ ,  $\cos[x]$ ,  $e^x$  and  $\log[x]$ .
- Why  $\log[x]$  is the natural logarithm and why  $e$  is the natural base for exponentials.
- What it means when  $f'[x]$  is positive or negative.
- Max-min.

#### Science and Math Experience

- Reading plots.
- Linear models.
- Drinking and driving.
- Japanese economy cars versus American big cars.
- Data analysis and interpolation.
- Data analysis of U.S. national debt and U.S. population in historical context including plots of yearly growth and the effect of immigration on the growth of the U.S. population.
- Cigarette smoking and lung cancer correlation.
- Global scale of quotients of functions studied by looking at dominant terms in the numerators and denominators.
- Recognition of exponential data, exponential data fit, carbon dating, credit cards, compound interest, effective interest rates, financial planning, decay of cocaine in the blood, underwater illumination, inflation.
- Relating the plots of  $f[x]$  and  $f'[x]$ .
- Using a plot of  $f'[x]$  to predict the plot of  $f[x]$ .
- Visualizing the limiting process by plotting  $f'[x]$  and  $(f[x+h] - f[x])/h$  on the same axes and seeing the plots coalesce as  $h$  closes in on 0.
- Spread of disease model.

#### Mathematics

- Instantaneous growth rates in context.
- The derivative as the instantaneous growth rate.
- Chain rule.
- Product rule as a consequence of the chain rule.
- Instantaneous percentage growth rate  $100 f'[x]/f[x]$  of a function  $f[x]$ .
- What it means when  $f'[x]$  is not equal to 0 for  $x = a$ .
- Why  $f[x]$  is not as big (or small) at  $x = a$  unless  $f'[a] = 0$ .
- The three differential equations:  $y'[x] = r y[x]$ ,  $y'[x] = r y[x] (1 - y[x]/b)$   $y'[x] = r y[x] + b$  and their solutions.
- The meaning of the parameters  $r$  and  $b$  in the three differential equations.
- Why it's often a good idea to view logistic growth as toned down exponential growth.

#### Science and Math Experience

- Another look at why exponential growth dominates power growth and why power growth dominates logarithmic growth.
- Logistic model of animal growth.
- The idea of linear dimension and using it to convert a model of animal height as a function of age to a model of animal weight as a function of age.
- Learning why the adolescent growth spurt is probably a mathematical fact instead of a biological accident.
- Compound interest.
- Making functions with prescribed instantaneous percentage growth rate.

- Why a good representative plot of a given function  $f[x]$  usually includes all  $x$ 's at which  $f[x] = 0$ .
- Max-Min in one or two variables.
- Using the derivative to get best least squares fit of data by smooth curves.
- Fitting of Space shuttle O-ring failure data as a function of temperature and using the result to explain why the Challenger disaster should have been predicted in advance.
- Data fit by lines and by Sine and Cosine waves.
- Optimal speed for salmon swimming up a river.
- Designing the least cost box to hold a given volume.
- Analysis of an oil slick at sea.
- How tall is the dog when it is growing the fastest?
- Analysis of what happens to  $x^k/e^x$  as  $x$  advances from 0 to Infinity.
- Models based on these differential equations.
- Why radio active decay is modeled by the differential equation  $y'[x] = r y[x]$ .
- Logistic versus exponential growth. Biological principles behind carbon dating.
- Growth of U.S. and world populations.
- Malthusian versus logistic models.
- Calculation of interest payments resulting from buying a car on time.
- Managing an inheritance.
- Wal-mart sales.
- Pollution elimination, data analysis.
- Speculating on why dogs and humans grow faster after their birth than they are at the instant of their birth, but horses grow fastest at the instant of their birth.
- Newton's law of cooling.
- Pressure altimeters.

### Mathematics

- The race track principles:
  - If  $f[a] = g[a]$  and  $f'[x] >= g'[x]$  for  $x >= a$ , then  $f[x] >= g[x]$  for  $x >= a$ .
  - If  $f[a] = g[a]$  and  $f'[x]$  is approximately equal to  $g'[x]$  for  $x >= a$ , then  $f[x]$  is approximately equal to  $g[x]$  for  $x >= a$ .
  - If  $f[a] = g[a]$  and  $f'[x] = g'[x]$  for  $x >= a$ , then  $f[x] = g[x]$  for  $x >= a$ .
- Euler's method of faking the plot of a function with a given derivative explained in terms of the race track principles.
- Euler's method of faking the plot of a the solution of a differential equation explained in terms of the race track principles.
- Plots of numerical approximations to solutions of first order differential equations. Qualitative analysis of first order differential equations and systems of first order differential equations.
- Parametric plotting of curves in two dimensions. Parametric plotting of curves and surfaces in three dimensions.
- Derivatives for curves given parametrically.

### Science and Math Experience

- Using the race track principle to explain why, as  $x$  advances from 0, the plots of solutions of  $y'[x] = r y[x]$  and  $y'[x] = ry[x] (1 - y[x]/b)$  will run close together in the case that  $y[0]$  is small relative to  $b$ .
- Why  $\sin[x] = x$  for  $x >= 0$  and related inequalities.
- Estimating how many accurate decimals of  $x$  are needed to get  $k$  accurate decimals of  $f[x]$ .
- The error function.
- Calculating accurate values of  $\log[x]$  and  $e^x$ .
- Analysis of the predator-prey model.
- Cycles in the predator-prey model.
- Drinking and driving model.
- Variable interest rates.
- Michaelis-Menten Drug equation.
- War games based on Lanchester war model including a simulation of the Battle of Iwo Jima.
- Harvesting in the logistic model. SIR epidemic model.
- The idea of chaos.
- Circular parameterization (polar coordinates) and other parameterizations.
- Projectile motion.
- Cams designed by sine and cosine wave fit.
- Predator-prey plotting.
- Parametric plotting of circles and ellipses.
- Elliptical orbits of planets and asteroids.

- Plotting of circles, tubes and horns centered on curves in three dimensions.
- Equilibrium populations in the predator-prey model.
- Modifications of the predator-prey model.
- The effect of poisoning predators with application to spraying insecticides.